14th National Convention on Statistics (NCS)
Crowne Plaza, Ortigas Center, Quezon City
01-03 October 1-3 2019

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by

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NONPARAMETRIC TEST OF HOMOGENEITY OF COVARIATE EFFECT IN A SPATIOTEMPORAL MODEL WITH VARYING FREQUENCIES

Daniel David M. Pamplona[1], Joseph Ryan G. Lansangan[2], and Erniel B. Barrios[3]

ABSTRACT

The estimation procedure of a spatiotemporal model with varying frequencies rely on the assumption of constant covariate effect across spatial units and time points. With this, a nonparametric procedure based on the bootstrap is proposed to validate the assumption of constant covariate effect across time points and locations. The test makes use of distances from limits of the estimated confidence interval of the covariate effect parameter. Simulation study shows that the size of the proposed test is consistent to be less than the nominal level of significance under different scenarios. The power of the test increases to 1.0 as alternative parameter values become more distant from the common value, and similarly, as the frequency of spatial units / time points with alternative parameter values increase. Sensitivity of the test decreased under the presence of large errors in the model, especially when the autocorrelation of error terms is high.

Keywords: spatiotemporal model, bootstrap, nonparametric, confidence interval

1. Introduction

A spatiotemporal model is a statistical representation of data sampled from specific locations over a period of time that characterizes their exogenous, spatial, and temporal dependencies (Cressie & Majure, 1997). The goal of this statistical modelling effort is spatiotemporal prediction, which is achieved by systematically modelling the relationship between a response variable and potential explanatory variables, while accounting for spatial dependence and temporal dependence simultaneously (Zhu, Huang, & Wu, 2005).

Malabanan and Barrios (2017) postulated a semiparametric spatiotemporal model, motivated by agricultural systems, with data measured at varying frequency, i.e., some variables are measured at higher time frequency than others. Unlike common approaches where aggregates or interpolations are used to create a set of covariates with the same time frequency, the differences in frequency is preserved to avoid loss of information. The proposed model optimizes utilization of information from variables measured at higher frequency by estimating its nonparametric effect on the response through the backfitting algorithm. The same model was used estimate corn yield per province in the Philippines, using vegetation index, amount of rainfall, and amount of fertilizers as predictors. Extensive simulation studies support the optimality of the model over simple generalized additive model (GAM) with aggregation of high frequency data. The simulation study, however, was designed to meet the assumptions of the model, namely: (1) constant nonparametric component effect across units and across time, (2) constant parametric component effect across units and across time, (3) constant neighborhood variable effect across units and across time, and (4) constant temporal effect across units. Hence, the gain in precision over GAM is not guaranteed if there are violations in the model.

It is possible that dynamic behavior of covariates may violate the model assumptions since the study of crop yield makes use of covariates which may vary across locations and time points. For instance, agricultural production throughout the year may be similar in provinces with

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the same rainfall conditions; soil quality and terrain tend to be similar in neighboring provinces, and agricultural programs (e.g. fertilizer application) vary over time for each province. Hence, the assumptions need to be tested first to facilitate correct interpretation of the estimates of Malabanan-Barrios model.

This paper aims to develop a nonparametric bootstrap-based hypothesis testing procedure to validate the assumption of the constant covariate effect in the spatiotemporal model with mixed frequencies. This validation procedure not only clarifies that model requirements are met, but also justifies the choice of the model for the specific data set used.

2. Spatiotemporal Model

An estimation procedure for an additive spatiotemporal model with mixed frequencies was proposed by Malabanan and Barrios (2017). The nonparametric effect of the higher frequency variable to the response was estimated using the backfitting algorithm. The following model was used:

$$Y_{it} = \sum_{k=1}^{K} f(X_{it_k}) + \beta Z_{it} + \gamma W_{it} + \varepsilon_{it}$$

$$\varepsilon_{it} = \rho\varepsilon_{i,t-1} + a_{it},\ |\rho| < 1,\ a_{it} \sim IID(0, \sigma_a^2),\ i = 1, \dots, n; t = 1, \dots, T; k = 1, \dots, K$$

where Y_{it} refers to the response in unit i at time t, X_{it_k} is the covariate measured at higher frequency of unit i at subtime k of time t, $f(\cdot)$ is any continuous function of X_{it_k}, Z_{it} is the covariate measured in the same frequency as response in spatial unit i at time t, W_{it} refers to variable the neighborhood system where spatial unit i belongs at time t, and ε_{it} is the error term for spatial unit i at time t, assumed to be independent and identically distributed (i.i.d), with mean zero.

The following steps constitute the algorithm used to generate the estimates of the spatiotemporal model with mixed frequencies:

Step 1. Estimate non-parametric component using spline smoothing. Then calculate $e_{it}^{(1)}$ to isolate the spatial and temporal components of the model.

$$e_{it}^{(1)} = y_{it} - \sum_{k=1}^{K} \widehat{f(X_{it_k})}$$

Step 2. Using $e_{it}^{(1)}$ from Step 1, estimate β and γ per spatial unit using least squares. The final estimates $\hat{\beta}$ and $\hat{\gamma}$ are taken as the average of $\hat{\beta}_i$ and $\hat{\gamma}_i$ per spatial unit. Calculate $e_{it}^{(2)}$ to isolate the temporal components of the model.

$$e_{it}^{(2)} = y_{it} - \left(\sum_{k=1}^{K} \widehat{f(X_{it_k})} + \hat{\beta} Z_{it} + \hat{\gamma} W_{it} \right)$$

Step 3. Using $e_{it}^{(2)}$ estimate temporal effect ρ. $e_{it}^{(2)}$ is modeled as AR(1) to estimate ρ_i per spatial unit. $\hat{\rho}$ is taken as the average of $\hat{\rho}_i$s.

Step 4. Calculate new observations

$$y_{it}^{new} = y_{it} - \hat{\rho} e_{i,t-1}^{(2)}$$

Step 5. Iterate process 1 to 4. Use the updated values of the dependent variable (y_{it}^{new}) for steps 1 and 2. To update the residuals in step 3, use the original values (y_{it}). Repeat step 4 using the updated estimates of the error terms and original values (y_{it}). The iteration continues until there is minimal change in the MSPE (<1%)

The proposed estimation procedure for spatiotemporal model with varying frequency of covariates produced better predictive ability over ordinary generalized additive model (GAM) under high rate of occurrence (K=12) of the higher frequency variable, and when the temporal correlation is high (ρ=0.9), regardless of the functional form of the more frequent covariate. These results support the value of the methodology in optimizing the use of unaggregated level of the higher frequency covariate in explaining the variability of the response. However, it was observed that the model did not show better predictions when the contribution of the higher frequency covariate to the response dominates the other predictors with lower frequency. Prediction errors of the model are larger when there is misspecification in the data (m=10) as compared to models without misspecification error (m=1).

3. Testing for Constant Covariate Effect

The following algorithm is proposed to test the assumptions of **constant covariate effect across locations** in a spatiotemporal model with mixed frequencies by Malabanan and Barrios (2017).

Let y_{it} be the response in location i and time point $t, i = 1, \dots, N$ and $t = 1, \dots, T$. Suppose β_i refers to the coefficient of covariate Z_{it} at location i, the following hypotheses are tested:

$$H_0: \beta_i = \beta \quad \forall i$$

$$H_1: \beta_i \neq \beta \quad for\ some\ i$$

where the null hypothesis of the test can be stated as: *all spatial units have the same covariate effect*, with the alternative hypothesis: *not all spatial units have the same covariate effect.*

Algorithm

1. Estimate the model $y_{it} = \sum_{k=1}^{K} f(x_{it_k}) + \beta z_{it} + \gamma w_{it} + \varepsilon_{it}$ by fitting the algorithm of Malabanan-Barrios model.
2. From the estimates in step 1, calculate the residual associated with the parameter of interest. For instance, to isolate the covariate effect, generate, generate the residuals by:

$$e_{it}^{\beta} = y_{it} - \left(\sum_{k=1}^{K} \widehat{f(X_{\iota t_k})} + \hat{\gamma} W_{it} \right)$$

3. Use the calculated residual as response $e_{it}^{\beta} = y_{it}^{*}$ to estimate the regression model:

$$y_{it}^{*} = \beta_i z_{it} + \varepsilon_{it}, \quad i = 1, . . ., N, \quad t = 1, \dots, T$$

4. In each spatial unit, generate k bootstrap samples of T pairs (z_{it}, y_{it}^{*}) from the original T pairs of observations. Let S_{ip} represent the p^{th} bootstrap sample from spatial unit i , where $p = 1, 2, . . ., k$.
5. For each bootstrap sample in spatial unit i, estimate model in step 3 using ordinary least squares.
6. Compute the bootstrap standard error of the estimated spatial parameter β_i for each spatial unit using the k bootstrap estimates $\hat{\beta}_{ip}^{*}$ generated in step 4.

$$\hat{\sigma}_{\hat{\beta}_i}^{*} = \left[\frac{1}{k-1} \sum_{p=1}^{k} \left(\hat{\beta}_{ip}^{*} - \overline{\hat{\beta}_{ip}^{*}} \right)^2 \right]^{1/2}, \qquad \overline{\hat{\beta}_{ip}^{*}} = \frac{1}{k} \sum_{p=1}^{k} \hat{\beta}_{ip}^{*}$$

7. Construct the $(1-\alpha) * 100\%$ normal-approximation confidence interval of the regression parameter β_i in each spatial unit. The confidence limits are as follows:

$$\widehat{\beta_\iota} \pm z_{\frac{\alpha}{2}} \hat{\sigma}_{\hat{\beta}_i}^{*}$$

8. Compute the test statistic for the test using the following procedure:
   i. Using the CIs generated from step 7, calculate the distance between the closest limits of all possible nonoverlapping CI pairs using the function d_l below. Let U and L represent the upper limit and lower limit respectively of the confidence interval.

$$d_l = \begin{cases} \min(|U_1 - L_2|, |L_1 - U_2|), & if\ CI\ pair\ does\ not\ overlap \\ 0, & otherwise \end{cases}$$

where $l = 1,2,\ldots,\binom{N}{2}$; there are $\binom{N}{2}$ pairs in total.

ii. The test statistic of the test is the mean of the distances:

$$D = \frac{1}{\binom{N}{2}} \sum_{l=1}^{\binom{N}{2}} d_l$$

9. Create the null distribution by replicating steps 1 to 8 using data with homogeneous covariate / spatial effects across spatial units. With 200 iterations, create an approximate distribution of the test statistic under the null hypothesis.
10. Generate the $(1-\alpha)^{th}$ percentile value (**critical value**) of the simulated null distribution of the test statistic in step 9.
11. Reject the null hypothesis if the test statistic in step 8 is greater than the **critical value** at $(\alpha * 100)$% level of significance.

The algorithm above can be modified to test the assumption of **constant covariate effect across time**. Instead of creating parameter estimates and confidence intervals for each spatial unit, the estimates and intervals can be generated for each of the T time points. The bootstrap resamples are generated from the N spatial units. And the testing procedure proceeds in the same manner, i.e., suppose β refers to the covariate effect, the following hypotheses are tested:

$$H_0: \beta_t = \beta \quad \forall t$$

$$H_1: \beta_t \neq \beta \;\; for\ some\ t$$

where the null hypothesis of the test can be stated as: *all time points have the same covariate effect*, with the alternative hypothesis: *not all time points have the same covariate effect*.

4. Simulation Study

The proposed nonparametric hypothesis testing procedures to evaluate the assumptions of the spatiotemporal model with mixed frequencies were evaluated using simulated data similar to the study of Malabanan and Barrios (2017). The response variable was generated using the model:

$$y_{it} = a \sum_{k=1}^{K} f(X_{it_k}) + \beta Z_{it} + \gamma W_{it} + \varepsilon_{it} \,,$$

$$\varepsilon_{it} = \rho \varepsilon_{i,t-1} + m a_{it} \,, \quad a_{it} \sim IID(0, \sigma_a).$$

where X_{it_k} is the higher temporal resolution covariate simulated from $U(0,1)$. The functional form of X_{it_k} is specified by $f(\cdot)$ which only take the linear form for this study. Z_{it} is the covariate with same frequency as the response simulated from $N(100,10)$, and W_{it} is the neighborhood variable generated from $Po(50)$. The errors were generated via AR Sieve in two scenarios with $\rho = 0.5$ and $\rho = 0.9$, with $a_{it} \sim IID(0,5)$. Error multiplier m, is known to induce bias in parametric modeling.

The simulated spatiotemporal data was generated based on different scenarios by varying: contribution of each model component to the response, sample size and length of time series, correlation of the error terms, error multiplier, and nature of covariate. *Table 1* shows the simulation boundaries used in the assessment of the proposed model and methodology. These parameter settings are a subset of the simulation scenarios created by Malabanan and Barrios (2017), but only the scenarios that were found to affect the performance of the estimation of procedure was included in this study to minimize computation time.

The weight of each simulated covariate, denoted by a, β, and γ, were set to have several scenarios that varies their contribution to the variability of the response. The covariates can have equal contribution, or one covariate dominates the rest.

Table 1. Simulation Boundaries

Parameter	Parameter Settings	Description
No. of subtime points per unit time	K=12	High temporal resolution
Contribution of each model component to the response (f(x)-z-w-e)	(30-30-30-10) (20-50-20-10) (20-20-50-10)	Equal contribution Dominating other covariate Dominating neighborhood covariate
Sample size and length of time series	(N=50, T=50) (N=50, T=80) (N=80, T=50)	Balanced data Longer time series than number of obs. More obs. than length of time series
Correlation of the error terms	ρ=0.5 ρ=0.9	Moderate Strong
Functional form of the $h(X_{it_k})$	$\sum(X_{it_k})$	Linear
Error multiplier	m=10 m=1	With misspecification error Without misspecification error
Nature of Covariate	ρ=0 ρ=0.5	No temporal autocorrelation With temporal autocorrelation

The covariates can assume temporal association since this is commonly encountered in spatiotemporal settings (e.g., precipitation index across time). These temporal characteristics were generated based on two characteristics: no temporal association, and with temporal correlation. The weights of the covariates were adjusted accordingly across temporal characteristics to maintain the desired model component contribution settings.

Simulation study also includes different sample sizes and lengths of time series. This was designed to assess the effect of varying lengths to the performance of the test when testing across time points and across locations.

4.1 Simulating Violation of Assumption

In this study, data was simulated with constant nonparametric and parametric effects across spatial units and time points – ideal for the proposed estimation procedure of Malabanan and Barrios (2017). The simulated response under the null hypothesis of constant model parameters across time and locations is given by the expression

$$y_{it} = a\sum_{k=1}^{K} f(X_{it_k}) + \beta Z_{it} + \gamma W_{it} + \varepsilon_{it}\,, \qquad \varepsilon_{it} = \rho\varepsilon_{i,t-1} + ma_{it}\,.$$

Datasets with violations in the assumptions of the model were also simulated. These violations were generated at varying degrees and at different rates of occurrence to capture a range of possible model violations. Specifically, three violations were simulated: (1) non-constant covariate effect, (2) non-constant neighborhood variable effect, and (3) non-constant temporal effect. These violations were generated by varying parameter values at randomly selected spatial units or time points. To illustrate, the simulated response under non-constant covariate effect across spatial units is given by:

$$y_{it} = a\sum_{k=1}^{K} f(X_{it_k}) + (\beta + v_i\beta)Z_{it} + \gamma W_{it} + \varepsilon_{it}\,, \qquad \varepsilon_{it} = \rho\varepsilon_{i,t-1} + ma_{it}$$

where v_i is a nonzero constant for some spatial unit i.

Varying the values of v_i also controls the percent difference of the non-constant parameter from the simulated “true” value. Likewise, response variable can be generated with non-constant neighborhood variable effect and nonconstant temporal effect by $(\gamma + v_i \gamma)$ and $(\rho + v_i \rho)$ respectively. The testing procedure were assessed using one model violation at a time.

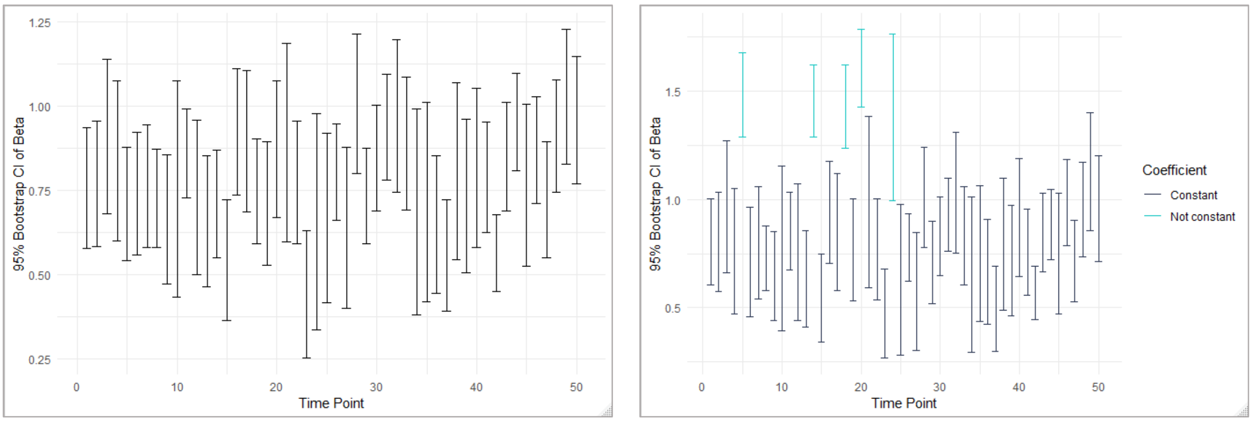
Table 2. Degree and Frequency of Violation in Model Assumption

Modification	Settings	Description / Remarks
Percent of spatial units / time points with different parameter values	10% 20%	Few Regular
Percent difference from "true" parameter value in simulation	$v = 0.3$ $v = 1.0$ $v = 2.0$	Weak Deviation Moderate Deviation Strong Deviation

The proposed tests were assessed across varying degrees of parameter difference and frequency of occurrence. As illustrated in Table 2, assumption violation was generated in 10% of spatial units or time points and in 20% of spatial units or time points, while maintaining constant parameter values to units or time points not selected. The differences of these parameters vary from 30% to 200% of the "true" parameter value in order to reflect weak to strong deviation. The simulated temporal parameter r, however, is not included in this range because of the restriction $|\rho| < 1$. Thus, percent difference for temporal parameter simulations was restricted to 30-80% of the "true" parameter value.

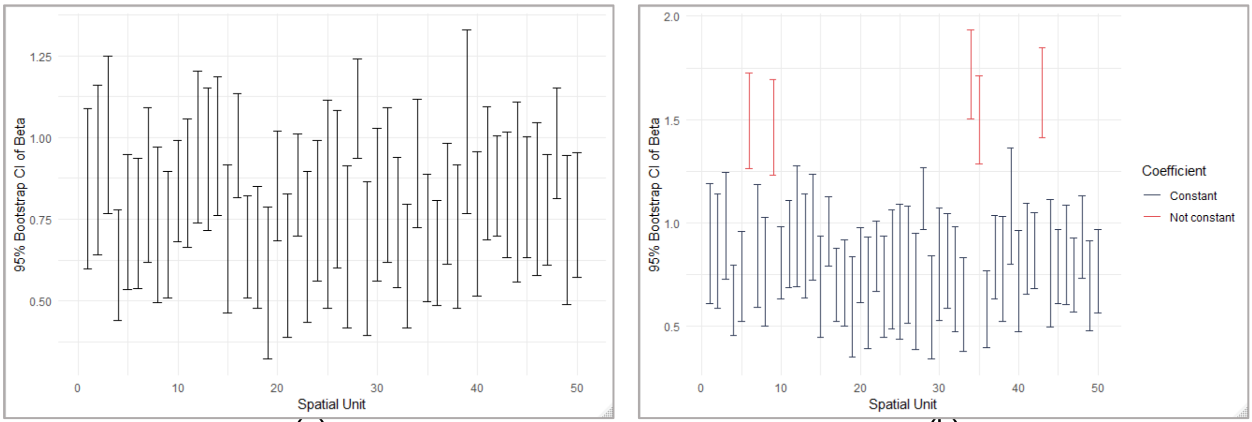
5. Results and Discussion

Performance of the test under varying data simulations were evaluated using its power and size. Power is the computed as the proportion of rejection under a false null hypothesis, while size is computed as the proportion of rejection under a true null hypothesis. For each scenario, two-hundred replicates were considered to calculate size and power at 5% level of significance. Covariate effect is characterized by the coefficient (β) associated with the covariates (Z) with similar frequency as the response variable. The proposed test evaluates if this coefficient attains homogeneity through time and location. The following figures below illustrate the behavior of the estimated bootstrap confidence intervals for constant and non-constant covariate effect:



(a) (b)

Figure 1. Sample Plot of 95% Bootstrap CI of β_t under (a) under $H_0: \beta_t = \beta = 0.79$; and (b) $H_1: \beta_t = \beta + v\beta$; $v = 1.0$ for 5 time points, $v = 0$ for 45 time points



(a) (b)

Figure 2. Sample Plot of 95% Bootstrap CI of β_t under (a) under $H_0\colon \beta_i = \beta = 0.79$; and (b) $H_1\colon\ \beta_i = \ \beta + v\beta$; $v = 1.0$ for 5 time points, $v = 0$ for 45 time points

5.1 Effect of Model Component Contributions to Testing Constant Covariate Effect

Table 3 and Table 4 illustrate the average empirical size and power of the test for different contributions of model component to the response. Three scenarios are considered: (1) equal contribution (30-30-30-10), (2) dominating covariate effect (20-50-20-10), and (3) dominating neighborhood variable effect (20-20-50-10). These scenarios were generated in order to identify the effect of varying model contributions to the estimation of covariate effect and as well as the performance of the proposed testing procedure.

The size of the test is observed to be unaffected by different component contributions and approximately remains under the nominal value of 0.05. The power of the test appears to be affected by two variables: the error multiplier (m), and the magnitude (v) of the model violation. Power is lowest when error of the model is high and magnitude of non-homogeneity in covariate effect is low, which appears common, given the difficulty in detecting non-homogeneity in covariate effects under small deviations. Sensitivity improves as the frequency of time points / locations with different covariate effect increase. This trend however could be misleading, since the higher frequency of deviation can mask the presence of non-homogeneity of coefficients. Among the scenarios of model component contributions, the power is slightly higher at (20-50-20-10), which is also expected given the variable being tested has the highest contribution to the response. The test performs at best when model error is controlled to a minimum and deviation in constancy of coefficient is almost twice the “true” value as indicated by power values close or equal to 1.00.

Table 3. Average empirical size and power in testing constant covariate effect across time points for different model component contributions

Error multiplier	Component Contribution *(f(x)-z-w-e)*	Size under $H_0\colon \beta_t = \beta$	Power against $H_1\colon\ \beta_t = \ \beta + v\beta$					
			5 time points			10 time points		
			$v = 0.3$	$v = 1$	$v = 2$	$v = 0.3$	$v = 1$	$v = 2$
m=1	(30-30-30-10)	0.051	0.141	0.858	1.000	0.232	0.946	1.000
	(20-50-20-10)	0.043	0.392	0.995	1.000	0.528	1.000	1.000
	(20-20-50-10)	0.065	0.073	0.605	0.974	0.101	0.749	0.995
m=10	(30-30-30-10)	0.053	0.051	0.064	0.083	0.057	0.063	0.136
	(20-50-20-10)	0.055	0.051	0.069	0.212	0.054	0.095	0.311
	(20-20-50-10)	0.043	0.045	0.057	0.062	0.047	0.059	0.078

Table 4. Average empirical size and power in testing constant covariate effect across locations for different model component contributions

Error multiplier	Component Contribution *(f(x)-z-w-e)*	Size under $H_0: \beta_t = \beta$	Power against $H_1: \beta_i = \beta + v\beta$					
			5 locations			5 locations		
			$v = 0.3$	$v = 1$	$v = 2$	$v = 0.3$	$v = 1$	$v = 2$
m=1	(30-30-30-10)	0.054	0.133	0.755	0.958	0.189	0.845	0.968
	(20-50-20-10)	0.051	0.302	0.921	0.982	0.433	0.963	0.985
	(20-20-50-10)	0.055	0.064	0.481	0.850	0.090	0.630	0.914
m=10	(30-30-30-10)	0.044	0.055	0.060	0.096	0.052	0.067	0.149
	(20-50-20-10)	0.047	0.049	0.071	0.196	0.053	0.105	0.300
	(20-20-50-10)	0.048	0.047	0.060	0.067	0.049	0.052	0.078

5.2 Effect of Error Correlation in Testing for Constant Covariate Effect

Error in the model is set to have AR(1) structure to characterize temporal dependencies in the response variable. Two cases are considered in the study: (1) moderate autocorrelation ($\rho = 0.5$), and (2) autocorrelation near non-stationarity ($\rho = 0.9$). Given that different error structures result to adjustments in estimating model coefficients, the performance of the proposed testing procedure is likewise expected to be different across different AR structures.

Size and power of the test are given in Tables 5 and 6. The test appears to be correctly sized and unaffected by different error correlations. On the other hand, power is observed to vary significantly according to two variables: the error multiplier (m), and the magnitude (v) of the model violation. In addition, sensitivity of the test is slightly better at error correlation of 0.5 as compared to 0.9, but comparable values are observed under high magnitude of non-homogeneity in coefficients. The test performs at best under minimum model prediction error and presence of high deviation from “true” coefficient value.

Table 5. Average empirical size and power in testing constant covariate effect across time for different error correlations

Error multiplier	Error correlation	Size under $H_0: \beta_t = \beta$	Power against $H_1: \beta_t = \beta + v\beta$					
			5 time points			10 time points		
			$v = 0.3$	$v = 1$	$v = 2$	$v = 0.3$	$v = 1$	$v = 2$
m=1	$\rho = 0.5$	0.054	0.285	0.959	1.000	0.425	0.990	1.000
	$\rho = 0.9$	0.042	0.118	0.680	0.983	0.148	0.807	0.997
m=10	$\rho = 0.5$	0.061	0.041	0.066	0.174	0.046	0.074	0.257
	$\rho = 0.9$	0.052	0.057	0.061	0.064	0.060	0.070	0.093

Table 6. Average empirical size and power in testing constant covariate effect across locations for different error correlations

Error multiplier	Error correlation	Size under $H_0: \beta_i = \beta$	Power against $H_1: \beta_i = \beta + v\beta$					
			5 locations			10 locations		
			$v = 0.3$	$v = 1$	$v = 2$	$v = 0.3$	$v = 1$	$v = 2$
m=1	$\rho = 0.5$	0.064	0.239	0.866	0.988	0.346	0.929	0.994
	$\rho = 0.9$	0.058	0.094	0.572	0.872	0.129	0.696	0.917
m=10	$\rho = 0.5$	0.042	0.063	0.083	0.168	0.055	0.093	0.260
	$\rho = 0.9$	0.037	0.037	0.045	0.072	0.048	0.056	0.091

5.3 Effect of Sample Size and Length of Time Series in Testing for Constant Covariate Effect

The sample size and length of time series is set to vary to represent datasets collected with unequal number of observations and time points. In this study, three cases are considered: (1) Equal sample size and time series length (N=T), (2) More observations than time points (N>T), and (3) less observations than time points (N<T).

In Tables 7 and 8, the size and power of the test are illustrated under different sample sizes and lengths of time series. Unlike other criteria being considered, the test performs slightly different when testing across time points as compared to testing across locations. In Table 7, power of the test is slightly higher when (N>T) when testing across time points, while in Table 8, power is higher when (N<T) when testing across locations. Sensitivity of the test is greatly affected by model prediction error and degree of model violation. Size is also observed to be under acceptable level of 0.05.

Table 7. Average empirical size and power in testing constant covariate effect across time for varying sample size and length of time series

Error multiplier	Sample size and length of time series	Size under $H_0: \beta_t = \beta$	Power against $H_1: \beta_t = \beta + v\beta$					
			5 time points			10 time points		
			$v = 0.3$	$v = 1$	$v = 2$	$v = 0.3$	$v = 1$	$v = 2$
m=1	N=T	0.062	0.165	0.772	0.985	0.235	0.877	0.997
	N>T	0.047	0.263	0.893	1.000	0.372	0.938	1.000
	N<T	0.048	0.178	0.793	0.990	0.254	0.880	0.998
m=10	N=T	0.045	0.047	0.066	0.104	0.047	0.063	0.132
	N>T	0.062	0.038	0.057	0.145	0.048	0.071	0.227
	N<T	0.050	0.062	0.068	0.107	0.064	0.083	0.167

Table 8. Average empirical size and power in testing constant covariate effect across locations for varying sample size and length of time series

Error multiplier	Sample size and length of time series	Size under $H_0: \beta_i = \beta$	Power against $H_1: \beta_i = \beta + v\beta$					
			5 locations			10 locations		
			$v = 0.3$	$v = 1$	$v = 2$	$v = 0.3$	$v = 1$	$v = 2$
m=1	N=T	0.042	0.128	0.653	0.901	0.200	0.744	0.924
	N>T	0.041	0.134	0.697	0.928	0.193	0.800	0.956
	N<T	0.055	0.237	0.808	0.961	0.319	0.894	0.988
m=10	N=T	0.046	0.045	0.056	0.099	0.050	0.067	0.151
	N>T	0.061	0.050	0.063	0.096	0.048	0.057	0.136
	N<T	0.056	0.056	0.073	0.164	0.056	0.100	0.239

5.4 Effect of Nature of Covariate in Testing Constant Covariate Effect

Given that the response variable is considered to have temporal dependencies, it follows that the associated covariates could potentially have an AR structure. In this study, two scenarios were created: (1) covariates have no temporal correlation ($\rho = 0$), and (2) covariates have temporal correlation ($\rho = 0.5$). The structure of the covariate is expected to change when assuming temporal dependency, hence performance of the test is evaluated across these scenarios.

It can be observed from Tables 9 and 10 that power of the test is slightly higher when testing across locations than testing across time points. Although power still approaches 1.0 as higher degrees of hon-homogeneity is imposed, this difference can be observed under lower frequency and degree of non-homogeneity of covariate effect. The test also appears correctly sized as indicated by values close or under 0.05.

Table 9. Average empirical size and power in testing constant covariate effect across time for different nature of covariate

Error multiplier	Nature of covariate	Size under $H_0: \beta_t = \beta$	Power against $H_1: \beta_t = \beta + v\beta$					
			5 time points			10 time points		
			$v = 0.3$	$v = 1$	$v = 2$	$v = 0.3$	$v = 1$	$v = 2$
m=1	$\rho = 0.0$	0.064	0.230	0.879	0.995	0.320	0.946	0.998
	$\rho = 0.5$	0.056	0.103	0.559	0.865	0.155	0.679	0.914
m=10	$\rho = 0.0$	0.050	0.053	0.069	0.151	0.049	0.087	0.234
	$\rho = 0.5$	0.038	0.047	0.059	0.089	0.054	0.062	0.117

Table 10. Average empirical size and power in testing constant covariate effect across locations for different nature of covariate

Error multiplier	Nature of covariate	Size under $H_0: \beta_i = \beta$	Power against $H_1: \beta_i = \beta + v\beta$					
			5 locations			10 locations		
			$v = 0.3$	$v = 1$	$v = 2$	$v = 0.3$	$v = 1$	$v = 2$
m=1	$\rho = 0.0$	0.056	0.237	0.878	0.999	0.341	0.946	1.000
	$\rho = 0.5$	0.047	0.167	0.761	0.984	0.233	0.851	0.997
m=10	$\rho = 0.0$	0.050	0.054	0.077	0.163	0.060	0.086	0.228
	$\rho = 0.5$	0.048	0.044	0.050	0.075	0.045	0.059	0.122

6. Conclusions

The proposed procedure in testing for constant covariate effect across time points and spatial units is correctly sized under the different scenarios simulated. Power of test follows an increasing pattern as frequency and degree of deviation from the true coefficient increase. In general, the test is sensitive under the presence of non-homogenous covariate effect that is at least twice the magnitude of the "true" coefficient, and when model errors are not too large. The frequency of time points / locations with different coefficients only slightly improves power, which is an advantageous finding, since the test is expected to detect non-constancy of coefficients regardless of the rate of occurrence of model violation.

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