A Structural Change of Distributed Lag Model in Bayesian Perspective

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14th NCS National Convention on Statistics
1-3 October 2019 | Crowne Plaza Manila Galleria
Organized by the Philippine Statistical System
Spearheaded by the Philippine Statistics Authority

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By

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Introduction

1. Economists are interested on when and how effects policy measures (*pertain to taxation, government budgets, the money supply and interest rates, labour market, national ownership, etc.*) will fully occur.

2. Dependent variables often react to changes in one or more of the explanatory variables only after a lapse of time. This delayed reaction suggests the inclusion of lagged explanatory variables into the specification of the model, resulting in a dynamic model.

Introduction

Lagged effects arise from different reasons.

1. Psychological – behavior is often based on inertia and habit, and expectation about future events are often based on past behavior.
2. Institutional – it takes time to respond to external events and certain rules lead to lagged responses.
3. Technical – production requires time, and durable goods last more than one period.

An economic example might be dividend payments by a corporation (Yt). This dependence is not only on earnings in the present period (Xt) but also on earnings in previous periods.

Introduction

The general form of a linear distributed lag model (DLM) is

$$Y_t = \phi + \sum_{i=0}^{\infty} \alpha_i X_{t-i} + \epsilon_t$$

where ϕ is a constant term, ϵ_t is the error term such that $\epsilon_t \sim N(0, \sigma_\epsilon^2),\ t = 1, 2, \ldots$, and any change in Xt will affect $E[Yt]$ in all the later periods.

The term α_i is the $i\text{th}$ reaction coefficient, and it is usually assumed that

$$\lim_{i \to \infty} \alpha_i = 0 \text{ and } \sum_{i=0}^{\infty} \alpha_i = \alpha < \infty.$$

Reviews about Structural Change in Time Series Models

1. **Supe, A. (1996)** – when modeling time-series data, parameters are assumed not to vary with time, but there are instances that model parameters also change after some specific time points.
2. **Western, B. et. al (2004)** – studied on a Bayesian model that treats the changepoint in a time series as a parameter to be estimated. In this model, inference for the regression coefficients reflects prior uncertainty about the location of the change point.
3. **Park, J. H. et. al (2007)** – introduced an efficient Bayesian approach to the multiple changepoint problem and discuss the utility of the Bayesian changepoint models in the context of generalized linear models.

Reviews about Structural Change in Time Series Models

4. Chaturvedia, A. et. al (2012) – considered the Bayesian analysis of a linear regression model involving structural change, which may occur either due to shift in disturbances precision or due to shift in regression parameters.

5. Cabactulan, F. (2014) – showed a Bayesian analysis procedure of estimating the parameter of the Koyck distributed lag model. His formulation of the posterior distribution of the parameters of the said model was done by kernel density identification on the resulting expression of the joint posterior distribution of the sample and the parameters.

The Distributed Lag Model

The general form of a linear distributed lag model (DLM) is

$$Y_t = \phi + \sum_{i=0}^{\infty} \alpha_i X_{t-i} + \epsilon_t.$$

Koyck suggested a simplification of the model and expressed as follows:

$$Z_t = \beta_0 + \beta X_t + u_t$$

where $Z_t = Y_t - \lambda Y_{t-1},\ \beta_0 = (1-\lambda)\phi$ and $u_t = \epsilon_t - \lambda\epsilon_{t-1}$.

The Distributed Lag Model

The structural change model to be considered is

$$Z_t = \begin{cases} \beta_0 + \beta_1 X_t + u_t, & t = 1, 2, \dots, \nu \\ \beta_0 + \beta_2 X_t + u_t, & t = \nu + 1, \dots, n \end{cases} \quad (1)$$

where $Z_t = Y_t - \lambda Y_{t-1},\ \beta_0 = (1-\lambda)\phi,\ \beta_2 = \beta_1 + \Delta,\ \Delta > 0$ and $u_t = \epsilon_t - \lambda\epsilon_{t-1}$.

Posterior Probability Distribution of the Break Point

Theorem: If the model (1) holds and ν, B, and τ are unknown, and if ν is uniformly distributed over $1, 2, \dots, n$, the joint prior distribution of B and τ is such that: the conditional distribution of B given τ is normal with mean B^* and precision matrix $\tau^{-1}\mathbf{I}$ $(\tau > 0)$ where $\mathbf{I}$ is a given $n \times n$ identity matrix and B^* is a 4×1 constant vector, the prior distribution of τ is gamma with parameters $a > 0$ and $b > 0$, and ν is independent of (B, τ) then the posterior distribution of ν given the sample observation $(\boldsymbol{X}, \boldsymbol{Z})$ is

$$\pi\big(\nu|(\boldsymbol{X},\boldsymbol{Z})\big) = K \cdot \begin{cases} |\boldsymbol{\Lambda}|^{-\frac{1}{2}}|\mathbf{U}|^{\frac{1}{2}}\left(\frac{2}{\mathbf{M}}\right)^{a+\frac{1}{2}}\Gamma(a+1/2), & 1 \le \nu \le n-1 \\ |\boldsymbol{\Lambda}|^{-\frac{1}{2}}|\mathbf{U_1}|^{\frac{1}{2}}\left(\frac{2}{\mathbf{M_1}}\right)^{a+\frac{1}{2}}\Gamma(a+1/2), & \nu = n \end{cases}$$

where

$$K = \frac{1}{\int |\boldsymbol{\Lambda}|^{-\frac{1}{2}}|\mathbf{U}|^{\frac{1}{2}}\left(\frac{2}{\mathbf{M}}\right)^{a+\frac{1}{2}}\Gamma(a+1/2)d\nu}$$

$$\mathbf{U} = \mathbf{X}'\boldsymbol{\Lambda}^{-1}\mathbf{X} + \mathbf{I}$$
$$\mathbf{V} = \mathbf{X}'\boldsymbol{\Lambda}^{-1}\mathbf{Z} + \mathbf{B}^*$$
$$\mathbf{W} = 2b + \mathbf{Z}'\boldsymbol{\Lambda}^{-1}\mathbf{Z} + \mathbf{B}^{*\prime}\mathbf{B}$$
$$\mathbf{M} = -\mathbf{V}'\boldsymbol{\Lambda}^{-1}\mathbf{V} + \mathbf{W}$$

Structural Change when $\sigma^2 = 1$

1. Exact detection is made only when β_2 is twice β_1.
2. Interval estimates (HPP near ν) consistently captures the break point. As change from β_1 to β_2 increases, point estimates improve while interval estimates give 100% capture of the break point.

TABLE 1. Simulation Results using the parameter values: $n = 10$, $\phi = 0.2$, $\lambda = 0.3$, $\beta_0 = 0.14$, $\beta_1 = 1$, and $\sigma^2 = 1$

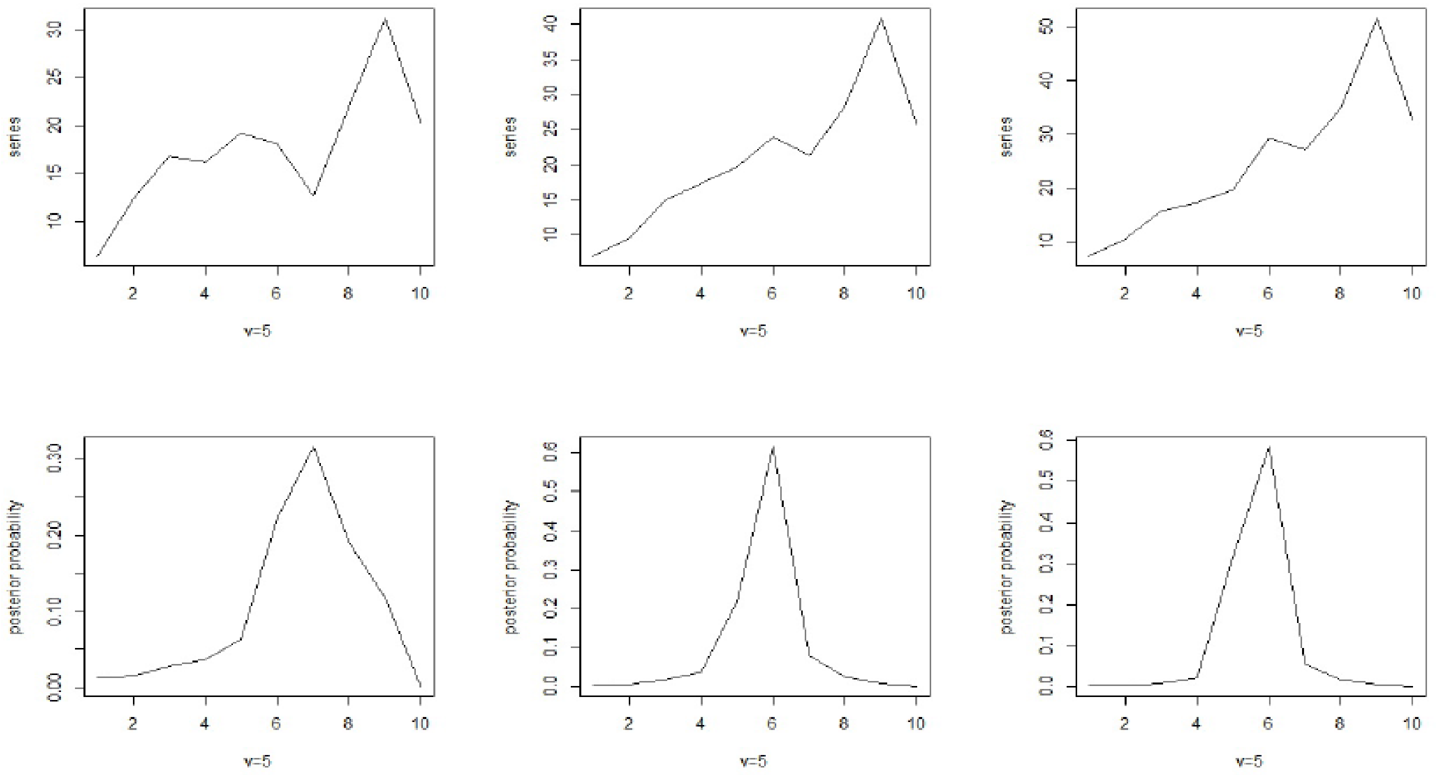
β_2	β_0^*	β_1^*	β_2^*	Break Point	HPP at ν	HPP near ν	Percentage near ν
1.2	0.14	1	1.2	5	0	49	98%
	0.14	1.3	1.4		0	48	96%
	0.14	1.5	1.6		0	49	98%
1.4	0.14	1	1.4	5	1	50	100%
	0.14	1.3	1.6		1	50	100%
	0.14	1.5	1.8		0	50	100%
1.6	0.14	1	1.6	5	0	50	100%
	0.14	1.3	1.8		1	50	100%
	0.14	1.5	2.0		1	50	100%
1.8	0.14	1	1.8	5	4	50	100%
	0.14	1.3	2.0		2	50	100%
	0.14	1.5	2.2		8	50	100%
2.0	0.14	1	2.0	5	15	50	100%
	0.14	1.3	2.2		18	50	100%
	0.14	1.5	2.4		13	50	100%

1. From the table below, $\nu = 6$ gives a probability of .4986 while $\nu = 5$ gives a posterior probability of .3935. Thus the point estimate is $\nu^* = 6$ but HPP near ν includes $\nu = 5$, the actual break point.
2. It is a pattern in the succeeding results that the point estimate HPP at ν tends to identify a value of ν which is one lag after the break point. This is because from the structure of the model, complete change in the model occurs after one lag.

TABLE 2. Posterior Distribution of ν for data based on $n = 10$, $\phi = 0.2$, $\lambda = 0.3$, $\beta_0^* = 0.14$, $\sigma^2 = 1$, and $\Delta = 1.0$

ν	1	2	3	4	5	6	7	8	9	10
pmf	.0013	.0025	.0085	.023	.3935	.4986	.0512	.0147	.0037	.0001

1. As β_2 goes farther away from β_1, the structural change in the model becomes easier to distinguish and the posterior probabilities tend to flock near ν, the break point.



(A) $\Delta = 0.2$ (B) $\Delta = 0.6$ (C) $\Delta = 1.0$

FIGURE 1. Plot of the simulated data and the corresponding posterior probability plot based on $n = 10$, $\beta_0^* = 0.14$, and $\sigma^2 = 1$

1. Detection at the exact break point is hardly attained when we increase the sample size n to 50.

TABLE 5. Simulation Results using the parameter values: $n = 50$, $\phi = 0.2$, $\lambda = 0.3$, $\beta_0 = 0.14$, $\beta_1 = 1$, and $\sigma^2 = 1$

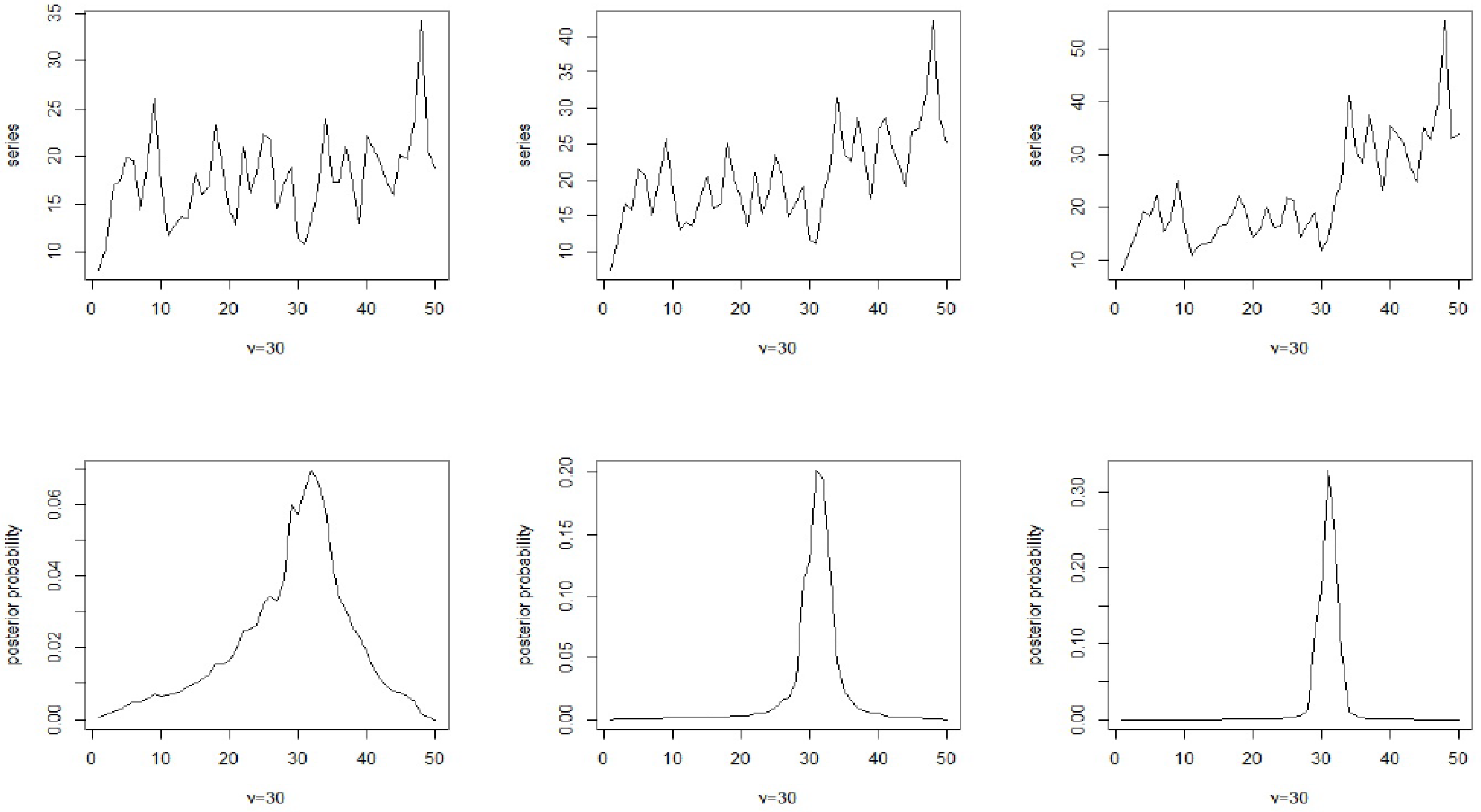
β_2	β_0^*	β_1^*	β_2^*	Break Point	HPP at ν	HPP near ν	Percentage near ν
1.2	0.14	1	1.2	30	0	43	86%
	0.14	1.3	1.4		0	44	88%
	0.14	1.5	1.6		0	40	80%
1.4	0.14	1	1.4	30	1	50	100%
	0.14	1.3	1.6		0	50	100%
	0.14	1.5	1.8		0	50	100%
1.6	0.14	1	1.6	30	0	50	100%
	0.14	1.3	1.8		0	50	100%
	0.14	1.5	2.0		0	50	100%
1.8	0.14	1	1.8	30	0	50	100%
	0.14	1.3	2.0		0	50	100%
	0.14	1.5	2.2		0	50	100%
2.0	0.14	1	2.0	30	1	50	100%
	0.14	1.3	2.2		0	50	100%
	0.14	1.5	2.4		0	50	100%

1. Break points are detected after one lag and can be seen in the posterior distribution of ν.
2. Full change can be detected after one lag because of the nature of the model which includes lagged variable. The interval estimate HPP near ν consistently captures the break point.

TABLE 12. Posterior Distribution of ν for data based on $n = 50$, $\phi = 0.2$, $\lambda = 0.3$, $\beta_0^* = 0.14$, $\sigma^2 = 1$, and $\Delta = 1.0$

ν	1	2	3	4	5	6	7	8	9	10
pmf	.0000	.0000	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0002
ν	11	12	13	14	15	16	17	18	19	20
pmf	.0003	.0003	.0003	.0004	.0004	.0005	.0005	.0007	.0007	.0008
ν	21	22	23	24	25	26	27	28	29	30
pmf	.0009	.0012	.0015	.0020	.0035	.0058	.0072	.0172	.1246	.1878
ν	31	32	33	34	35	36	37	38	39	40
pmf	.3058	.2099	.0889	.0162	.0072	.0046	.0025	.0017	.0015	.0010
ν	41	42	43	44	45	46	47	48	49	50
pmf	.0007	.0005	.0004	.0004	.0003	.0002	.0002	.0001	.0000	.0000

1. As β_2 goes farther away from β_1, the structural change in the model becomes easier to distinguish and the posterior probabilities tend to flock near ν, the break point.



(A) $\Delta = 0.2$ (B) $\Delta = 0.6$ (C) $\Delta = 1.0$

FIGURE 4. Plot of the simulated data and the corresponding posterior probability plot based on $n = 50$, $\beta_0^* = 0.14$, and $\sigma^2 = 1$

Structural Change when $\sigma^2 = 2$.

1. The point estimate of the break point ν (HPP at ν) hardly detects the simulated break point when $\sigma^2 = 2$ as compared to the detection when $\sigma^2 = 1$.
2. However, the highest posterior probability is attained after one lag, so the interval estimate will contain the simulated break point.

TABLE 6. Simulation Results using the parameter values: $n = 10$, $\phi = 0.2$, $\lambda = 0.3$, $\beta_0 = 0.14$, $\beta_1 = 1$, and $\sigma^2 = 2$

β_2	β_0^*	β_1^*	β_2^*	Break Point	HPP at ν	HPP near ν	Percentage near ν
1.2	0.14	1	1.2	5	0	44	88%
	0.14	1.3	1.4		0	39	78%
	0.14	1.5	1.6		0	45	90%
1.4	0.14	1	1.4	5	0	49	98%
	0.14	1.3	1.6		0	49	98%
	0.14	1.5	1.8		0	50	100%
1.6	0.14	1	1.6	5	0	50	100%
	0.14	1.3	1.8		0	50	100%
	0.14	1.5	2.0		0	50	100%
1.8	0.14	1	1.8	5	0	50	100%
	0.14	1.3	2.0		0	50	100%
	0.14	1.5	2.2		0	50	100%
2.0	0.14	1	2.0	5	0	50	100%
	0.14	1.3	2.2		0	50	100%
	0.14	1.5	2.4		0	50	100%

1. When the error variance is increased from $\sigma^2 = 1$ to $\sigma^2 = 2$, the detection of the exact value of the break point is consistently not attained. However, the detection is one lag after the exact break point. This implies that the change in variance from 1 to 2 does not change the fact that the detection is attained after one lag of the point ν.

TABLE 9. Simulation Results using the parameter values: $n = 50$, $\phi = 0.2$, $\lambda = 0.3$, $\beta_0^* = 0.14$, $\beta_1 = 1$, and $\sigma^2 = 2$

β_2	β_0^*	β_1^*	β_2^*	Break Point	HPP at ν	HPP near ν	Percentage near ν
1.2	0.14	1	1.2	30	0	17	34%
	0.14	1.3	1.4		0	18	36%
	0.14	1.5	1.6		0	22	44%
1.4	0.14	1	1.4	30	0	48	96%
	0.14	1.3	1.6		0	46	92%
	0.14	1.5	1.8		0	45	90%
1.6	0.14	1	1.6	30	0	50	100%
	0.14	1.3	1.8		0	50	100%
	0.14	1.5	2.0		0	50	100%
1.8	0.14	1	1.8	30	0	50	100%
	0.14	1.3	2.0		0	50	100%
	0.14	1.5	2.2		0	50	100%
2.0	0.14	1	2.0	30	0	50	100%
	0.14	1.3	2.2		0	50	100%
	0.14	1.5	2.4		0	50	100%

Posterior Distributions

Posterior distribution of ν for data based on $\phi = 0.2$, $\lambda = 0.3$, $\boldsymbol{\beta_0^* = 0.14}$, $\boldsymbol{\sigma^2 = 1}$, and $\Delta = 1.0$

ν	p.m.f $n=10$	p.m.f $n=15$	ν	p.m.f $n=30$	ν	p.m.f $n=50$	ν	p.m.f
1	0.0013	0.0008	1	0.00004	1	0.00002	31	0.3058
2	0.0025	0.0018	2	0.00007	2	0.00003	32	0.2099
3	0.0085	0.0045	3	0.0001	3	0.00006	33	0.0889
4	0.0230	0.0088	4	0.0002	4	0.00008	34	0.0162
5	0.3935	0.0171	5	0.0003	5	0.00010	35	0.0072
6	0.4986	0.0396	6	0.0005	6	0.00014	36	0.0046
7	0.0512	0.0738	7	0.0006	7	0.00015	37	0.0025
8	0.0147	0.4533	8	0.0009	8	0.00017	38	0.0017
9	0.0037	0.3597	9	0.0027	9	0.00023	39	0.0015
10	0.0001	0.0228	10	0.0028	10	0.00024	40	0.0010
11		0.0089	11	0.0032	11	0.00026	41	0.00072
12		0.0045	12	0.0047	12	0.00028	42	0.00053
13		0.0028	13	0.0084	13	0.00032	43	0.00042
14		0.0014	14	0.0276	14	0.00036	44	0.00036
15		0.0001	15	0.3029	15	0.00041	45	0.00028
16			16	0.5339	16	0.00045	46	0.00022
17			17	0.0878	17	0.00052	47	0.00015
18			18	0.0121	18	0.00065	48	0.00006
19			19	0.0041	19	0.00070	49	0.00003
20			20	0.0022	20	0.00076	50	0.0000
21			21	0.0015	21	0.00088		
22			22	0.0010	22	0.0012		
23			23	0.0007	23	0.0015		
24			24	0.0005	24	0.0020		
25			25	0.0004	25	0.0035		
26			26	0.0003	26	0.0058		
27			27	0.00018	27	0.0072		
28			28	0.00012	28	0.0172		
29			29	0.00004	29	0.1246		
30			30	0.0000	30	0.1878		

Posterior Distributions

Posterior distribution of ν for data based on $\phi = 0.2$, $\lambda = 0.3$, $\boldsymbol{\beta_0^* = 0.20}$, $\boldsymbol{\sigma^2 = 1}$, and $\Delta = 1.0$

ν	p.m.f $n = 10$	p.m.f $n = 15$	ν	p.m.f $n = 30$	ν	p.m.f $n = 50$	ν	p.m.f
1	0.0011	0.00066	1	0.00006	1	0.00002	31	0.28419
2	0.0024	0.00146	2	0.00011	2	0.00003	32	0.23509
3	0.0089	0.00357	3	0.00021	3	0.00006	33	0.11066
4	0.0299	0.00706	4	0.00031	4	0.00007	34	0.01837
5	0.4392	0.01490	5	0.00047	5	0.00010	35	0.00771
6	0.4572	0.04309	6	0.00069	6	0.00013	36	0.00494
7	0.0456	0.10433	7	0.00082	7	0.00014	37	0.00264
8	0.01244	0.63572	8	0.00121	8	0.00016	38	0.00163
9	0.0031	0.16236	9	0.00345	9	0.00022	39	0.00133
10	0.0001	0.01374	10	0.00372	10	0.00023	40	0.00095
11		0.00587	11	0.00416	11	0.00025	41	0.00067
12		0.00360	12	0.00598	12	0.00027	42	0.00051
13		0.00234	13	0.01030	13	0.00031	43	0.00041
14		0.00124	14	0.02916	14	0.00035	44	0.00034
15		0.00007	15	0.30199	15	0.00039	45	0.00027
16			16	0.53417	16	0.00043	46	0.00021
17			17	0.07707	17	0.00048	47	0.00014
18			18	0.01240	18	0.00059	48	0.00005
19			19	0.00466	19	0.00064	49	0.00003
20			20	0.00271	20	0.00070	50	0.00000
21			21	0.00196	21	0.00080		
22			22	0.00135	22	0.00110		
23			23	0.00094	23	0.00133		
24			24	0.00073	24	0.00173		
25			25	0.00053	25	0.00294		
26			26	0.00036	26	0.00509		
27			27	0.00026	27	0.00678		
28			28	0.00018	28	0.01748		
29			29	0.00005	29	0.11625		
30			30	0.00001	30	0.17075		

Posterior Distributions

Posterior distribution of ν for data based on $\phi = 0.2$, $\lambda = 0.3$, $\boldsymbol{\beta_0^* = 0.14, \sigma^2 = 2}$, and $\Delta = 1.0$

ν	p.m.f $n = 10$	p.m.f $n = 15$	ν	p.m.f $n = 30$	ν	p.m.f $n = 50$	ν	p.m.f
1	0.00164	0.00056	1	0.00011	1	0.00006	31	0.18614
2	0.00329	0.00133	2	0.00021	2	0.00011	32	0.23440
3	0.01162	0.00298	3	0.00042	3	0.00019	33	0.18348
4	0.02929	0.00457	4	0.00060	4	0.00026	34	0.05291
5	0.12394	0.00773	5	0.00089	5	0.00036	35	0.02228
6	0.70864	0.01447	6	0.00129	6	0.00046	36	0.01315
7	0.09001	0.02554	7	0.00151	7	0.00050	37	0.00686
8	0.02495	0.10258	8	0.00220	8	0.00058	38	0.00468
9	0.00643	0.79545	9	0.00512	9	0.00079	39	0.00397
10	0.00019	0.02852	10	0.00523	10	0.00080	40	0.00313
11		0.00894	11	0.00574	11	0.00085	41	0.00236
12		0.00417	12	0.00781	12	0.00092	42	0.00173
13		0.00213	13	0.01222	13	0.00099	43	0.00132
14		0.00097	14	0.02585	14	0.00109	44	0.00109
15		0.00005	15	0.10776	15	0.00122	45	0.00087
16			16	0.48228	16	0.00136	46	0.00069
17			17	0.26522	17	0.00150	47	0.00047
18			18	0.04482	18	0.00184	48	0.00018
19			19	0.01252	19	0.00199	49	0.00009
20			20	0.00609	20	0.00218	50	0.00000
21			21	0.00395	21	0.00249		
22			22	0.00255	22	0.00323		
23			23	0.00176	23	0.00365		
24			24	0.00132	24	0.00471		
25			25	0.00095	25	0.00750		
26			26	0.00065	26	0.01111		
27			27	0.00048	27	0.01295		
28			28	0.00032	28	0.02468		
29			29	0.00009	29	0.08829		
30			30	0.00001	30	0.10348		

Posterior Distributions

Posterior distribution of ν for data based on $\phi = 0.2$, $\lambda = 0.3$, $\boldsymbol{\beta_0^* = 0.20}$, $\boldsymbol{\sigma^2 = 2}$, and $\Delta = 1.0$

ν	p.m.f $n = 10$	p.m.f $n = 15$	ν	p.m.f $n = 30$	ν	p.m.f $n = 50$	ν	p.m.f
1	0.00138	0.00089	1	0.00016	1	0.00004	31	0.24466
2	0.00263	0.00186	2	0.00030	2	0.00007	32	0.23199
3	0.00768	0.00389	3	0.00061	3	0.00014	33	0.11755
4	0.01955	0.00642	4	0.00088	4	0.00019	34	0.02818
5	0.14499	0.01226	5	0.00135	5	0.00026	35	0.01223
6	0.65967	0.02895	6	0.00198	6	0.00032	36	0.00781
7	0.12922	0.05847	7	0.00228	7	0.00034	37	0.00431
8	0.02755	0.28530	8	0.00346	8	0.00038	38	0.00292
9	0.00715	0.55427	9	0.00812	9	0.00050	39	0.00244
10	0.00017	0.02755	10	0.00801	10	0.00051	40	0.00187
11		0.01013	11	0.00839	11	0.00054	41	0.00143
12		0.00537	12	0.01138	12	0.00059	42	0.00109
13		0.00303	13	0.01784	13	0.00065	43	0.00088
14		0.00152	14	0.03836	14	0.00071	44	0.00075
15		0.00007	15	0.13337	15	0.00081	45	0.00061
16			16	0.42619	16	0.00091	46	0.00048
17			17	0.23543	17	0.00103	47	0.00032
18			18	0.05502	18	0.00131	48	0.00013
19			19	0.01754	19	0.00144	49	0.00007
20			20	0.00910	20	0.00159	50	0.00000
21			21	0.00644	21	0.00186		
22			22	0.00436	22	0.00249		
23			23	0.00298	23	0.00294		
24			24	0.00229	24	0.00384		
25			25	0.00165	25	0.00682		
26			26	0.00107	26	0.01217		
27			27	0.00076	27	0.01442		
28			28	0.00051	28	0.02859		
29			29	0.00013	29	0.10998		
30			30	0.00002	30	0.14483		

Summary and Conclusion

1. In a Koyck distributed lag model which undergoes structural change, the highest posterior probability is generally attained at $\nu + 1$, which is one lag after the simulated break point. This is due to the fact that the model includes lagged variable that gives delayed reaction to the dependent variable.

2. The interval estimate HPP near ν consistently and efficiently captures the real value of the break point in the interval $\mathrm{HPP_t} \pm 5\%$ of n.

3. When the error variance is increased from $\sigma^2 = 1$ to $\sigma^2 = 2$, the detection of the exact value of the break point is consistently not attained. However, the detection is one lag after the exact break point. This implies that the change in variance from 1 to 2 does not change the fact that the detection is attained after one lag of the point.

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