BAYESIAN ESTIMATION OF THE GJR-GARCH (*p, q*) MODEL

By

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Presented by

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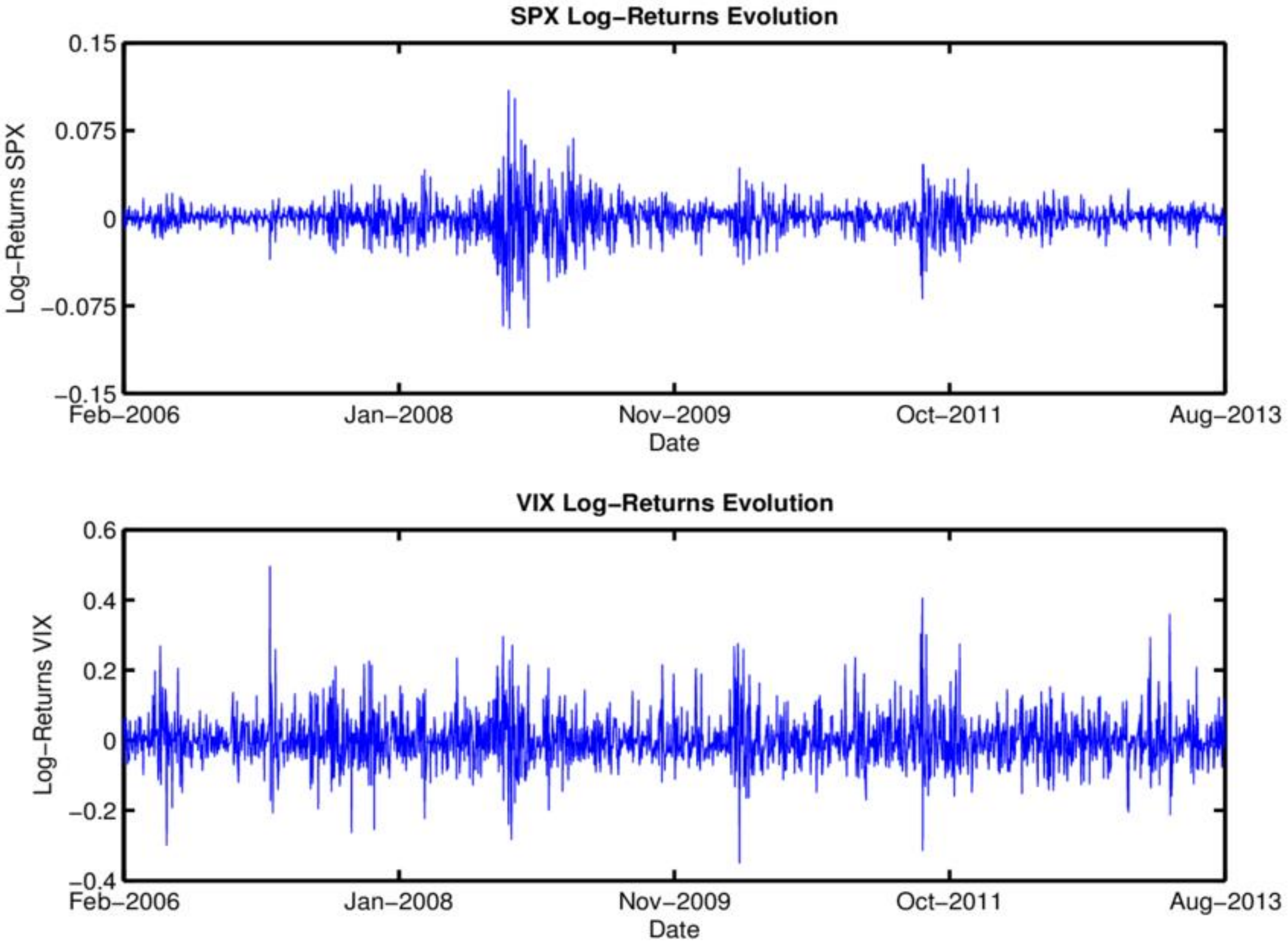
- **Homoscedasticity** - the variance of the error terms is constant across all observations.

- **Heteroscedasticity** - the variance of the error terms changes over time or across observations.

- *Volatility* is the rate at which the price of a security increases or decreases. It is measured by calculating the standard deviation of the annualized returns over a given period of time.

Volatility Example:



- Robert Engle in 1982 developed the Autoregressive Conditional Heteroskedastic (ARCH) model.

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-1}{}^2$$

- Tim Bollerslev(1986) called Generalized Autoregressive Conditional Heteroskedastic (GARCH) model.

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-1}{}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$$

- Nelson in 1991 presented an alternative way to the GARCH model by modifying it to Exponential GARCH (EGARCH) model.

$$\log(h_t) = \alpha_0 + \beta \log(h_{t-1}) + \alpha \left| \frac{\varepsilon_{t-1}}{(h_t)^{1/2}} \right| + \gamma \frac{\varepsilon_{t-1}}{(h_t)^{1/2}}$$

Glosten, Jagannathan and Runkle (1993) GJR-GARCH model:

$$h_t = \alpha_0 + \sum_{i=1}^{q} \left(\alpha_i \mathbb{I}_{\{u_{t-i} \geq 0\}} + {\alpha_i}^* \mathbb{I}_{\{u_{t-i} < 0\}} \right) {u_{t-i}}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$$

where $\alpha_0 > 0$;

$\alpha_i \geq 0 \ (i = 1, \ldots, q)$;

${\alpha_i}^* \geq 0 \ \ (i = 1, \ldots, q)$; and

$\beta_j \geq 0 \ (j = 1, \ldots, p)$

- Maximum Likelihood estimates are known to be statistically effecient but due to its complexity of the computations, maximum likelihood has become less practical use in estimation.

- According to Engle (1982), the normality assumption of the error terms may not be appropriate in some applications since heavy tails are commonly observed in economic and financial data.

Bayesian Estimation

- θ is a random variable (has an unknown distribution)
- Inference uses both data and prior information

$$Posterior = \frac{Likelihood\ x\ prior}{\int (Likelihood\ x\ prior)}$$

Frequentist

- θ is fixed and unknown
- Inference uses data only

OBJECTIVES:

- To provide estimates of the GJR-GARCH (*p, q*) model with student-*t* distribution error.

- To compare the classical Maximum Likelihood estimates to the estimates of Bayesian Estimation using GJR-GARCH (*p, q*) with student-*t* error distribution.

SIMULATION RESULT

- The parameters $\alpha_i (i = 0,1, \ldots , q)$, $\alpha_i{}^* (i = 1, \ldots , q)$, $\beta_j (j = 1, \ldots , p)$ and v was preset to obtain the simulated data with sample sizes 100, 500, 1000, 2500 and 5000.

- For each MCMC simulation, two chains were run with 5,000 iterations for each chain. The burn-in period was set to 2,500 for each chain. This implies that the first 2,500 iterations from each chain were disregarded. Hence, a total of 5,000 values of the chain were considered as samples from the true posterior density of the parameters.

Bayesian and Classical estimates of Student-*t* Error Distribution

- The sample size considered were $n = 500$

	Parameter	True Value	Mean Estimate		MSE	
			Bayesian	MLE	Bayesian	MLE
GJR-GARCH(2,2)	α_0	0.02	**0.01924274**	0.13637984	**0.000000581**	0.06398750
	α_1	0.06	0.11737537	**0.0491917**	0.003293397	**0.00239422**
	α_2	0.04	0.10452251	**0.04209486**	0.004164276	**0.00193570**
	${\alpha_1}^*$	0.04	**0.10442038**	0.1132863	**0.004151485**	0.31536200
	${\alpha_2}^*$	0.05	**0.10028100**	0.1467286	**0.002529252**	0.42630420
	β_1	0.1	**0.16316498**	0.1981398	**0.003991120**	0.08145582
	β_2	0.2	0.27862504	**0.2382006**	**0.006183625**	0.08373277
	v	7	**7.02848963**	7.5999792	**0.000847804**	3.48191700

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- The sample size considered were $n = 5000$

	Parameter	True Value	Mean Estimate		MSE	
			Bayesian	MLE	Bayesian	MLE
GJR-GARCH(2,2)	α_0	0.02	**0.0174743**	0.02986668	**0.00000637**	0.00070621
	α_1	0.06	0.03960772	**0.06190507**	0.00041603	**0.0004070**
	α_2	0.04	**0.0429258**	0.03347098	**0.00000873**	0.00059193
	$\alpha_1{}^*$	0.04	**0.0431264**	0.06588245	**0.00000997**	0.03590558
	$\alpha_2{}^*$	0.05	**0.0383348**	0.13779299	**0.00013624**	0.21125680
	β_1	0.1	**0.1230772**	0.20761569	**0.00053374**	0.04678681
	β_2	0.2	0.3159739	**0.12274574**	**0.01345109**	0.03560983
	v	7	**7.0002337**	7.07707675	**0.00000345**	0.46705380

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GJR-GARCH(2,2)	α_0	0.02	**0.0174743**	0.02986668	**0.00000637**	0.00070621
	α_1	0.06	0.03960772	**0.06190507**	0.00041603	**0.0004070**
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	β_1	0.1	**0.1230772**	0.20761569	**0.00053374**	0.04678681
	β_2	0.2	0.3159739	**0.12274574**	**0.01345109**	0.03560983
	v	7	**7.0002337**	7.07707675	**0.00000345**	0.46705380

Bayesian and Classical estimates of Student-*t* Error Distribution

- The sample size considered were $n = 100$

	Parameter	True Value	Mean Estimate		MSE	
			Bayesian	MLE	Bayesian	MLE
GJR-GARCH(1,2)	α_0	**0.02**	**0.02284653**	0.12959452	**0.00000812**	0.0574080
	α_1	0.05	0.20776956	**0.04999872**	0.02489699	**0.0073399**
	α_2	0.08	0.25090539	**0.08268486**	0.02921679	**0.0130662**
	${\alpha_1}^*$	0.07	0.11608007	**0.04052928**	**0.00212457**	0.4514425
	${\alpha_2}^*$	0.05	**0.16612480**	0.18013963	**0.01348762**	0.4101193
	β_1	0.4	**0.36421032**	0.49100089	**0.00128541**	0.1506002
	v	7	**7.1998996**	8.12046272	**0.0401654**	7.0418080

Bayesian and Classical estimates of Student-*t* Error Distribution

- The sample size considered were $n = 2500$

GJR-GARCH(1,2)	α_0	**0.02**	**0.0268218**	0.03964822	**0.00004653**	0.00279226
	α_1	0.05	0.07580281	**0.04689258**	**0.00066615**	0.00071402
	α_2	0.08	0.05499345	**0.08547741**	**0.00062571**	0.00143607
	${\alpha_1}^*$	0.07	**0.0573692**	0.23549212	**0.00015976**	0.20274810
	${\alpha_2}^*$	0.05	0.06235970	**0.05499876**	**0.00015305**	0.09042198
	β_1	0.4	0.25001016	**0.40068787**	**0.02249732**	0.03023907
	v	7	**7.1672520**	7.17929480	**0.02798094**	0.85707670

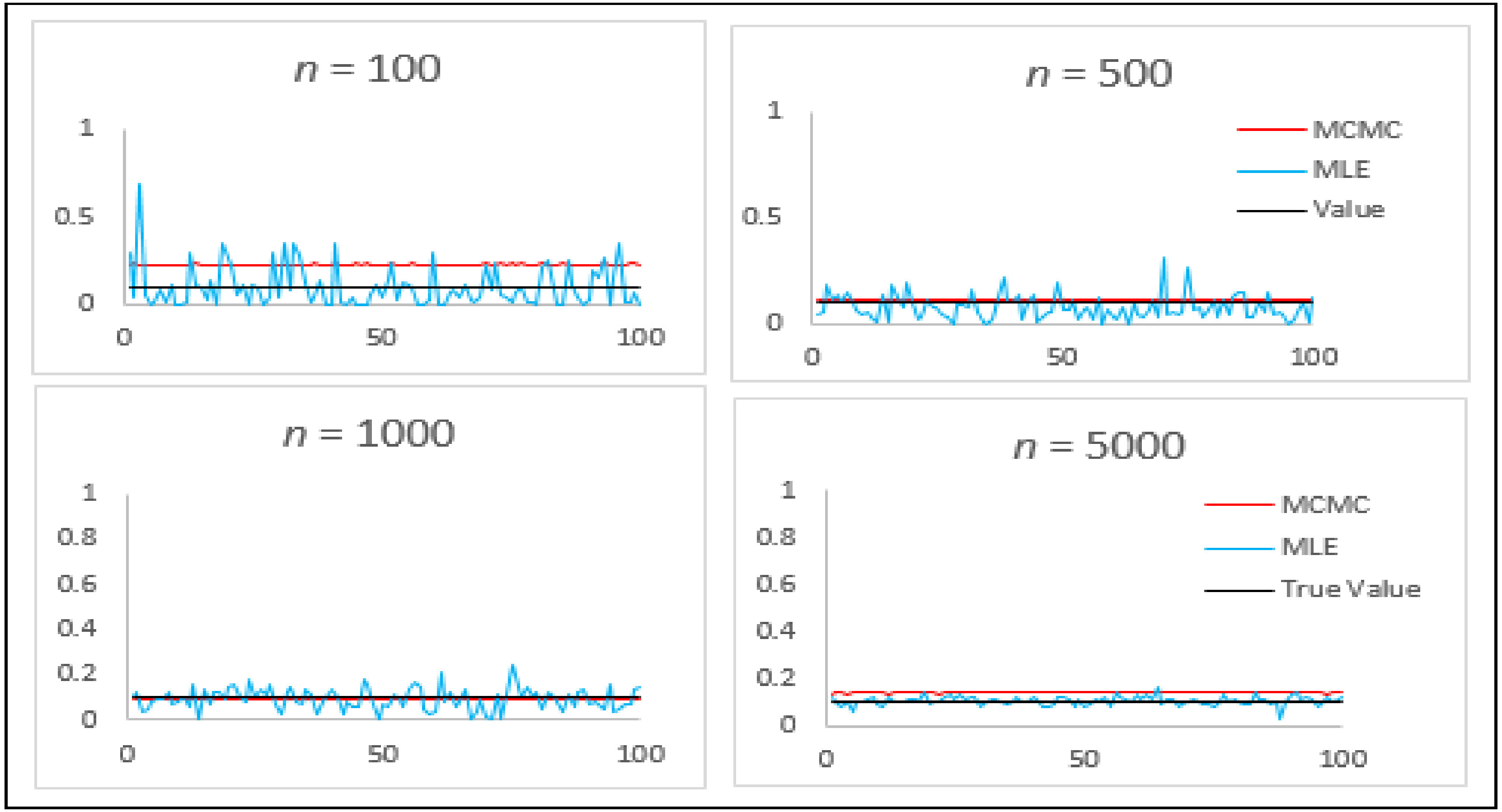

Figure 1 GJR-GARCH(1,1) Bayesian and Classical Estimates for α_1

Conclusion

The study shows that the Bayesian estimation of the GJR-GARCH (*p, q*) model with student-*t* distribution provides a better estimates than the classical Maximum Likelihood estimation.

End of Presentation