BAYESIAN ESTIMATION OF A-PARCH MODEL: AN APPLICATION TO JOLLIBEE FOOD CORPORATION STOCK MARKET

By

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Presented by

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BAYESIAN ESTIMATION OF A-PARCH MODEL WITH STUDENT'S *t*-DISTRIBUTED INNOVATIONS

The Asymmetric Power Autoregressive Heteroscedasticity (A-PARCH(*p*,*q*)) model of the error terms y_t with Student's t-distributed innovations ϵ_t can be written using data augmentation as

$$y_t = \epsilon_t (\sigma h_t)^{\frac{1}{2}}$$

$$\epsilon_t \sim N(0,1)$$

$$\omega_t \sim IG\left(\frac{\upsilon}{2}, \frac{\upsilon}{2}\right)$$

$$\sigma = \frac{\upsilon - 2}{\upsilon}$$

$$h_t^{\frac{\delta}{2}} = \alpha_0 + \sum_{i=1}^{p} \alpha_i (|y_{t-i}| - \gamma_i y_{t-i})^{\delta} + \sum_{j=1}^{q} \beta_j h_{t-j}^{\frac{\delta}{2}}$$

The following are the proposed priors for $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\gamma}$:

$$p(\boldsymbol{\alpha}) \propto N_{p+1}(\boldsymbol{\alpha}|\boldsymbol{\mu}_{\alpha}, \Sigma_{\boldsymbol{\alpha}})I_{(\boldsymbol{\alpha}>0)}$$

$$p(\boldsymbol{\beta}) \propto N_{q}(\boldsymbol{\beta}|\boldsymbol{\mu}_{\boldsymbol{\beta}}, \Sigma_{\boldsymbol{\beta}})I_{(\boldsymbol{\beta}\geq 0)}$$

$$p(\boldsymbol{\gamma}) \propto N_{p}(\boldsymbol{\gamma}|\boldsymbol{\mu}_{\boldsymbol{\gamma}}, \Sigma_{\boldsymbol{\gamma}})I_{(|\boldsymbol{\gamma}|<1)}.$$

The approximate likelihood function of $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$ is

$$L(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}|\boldsymbol{y}) = |\Lambda|^{-\frac{1}{2}} exp\left\{-\frac{1}{2}\boldsymbol{z}'\Lambda\boldsymbol{z}\right\}.$$

The construction of the posterior (proposal) densities for $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\gamma}$ will be based on this likelihood function.

Based on the approximate likelihood function of $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})$ and the prior densities of the parameters $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\gamma}$, and using the Bayes' theorem, the following are the posterior (proposal) densities of the parameters $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\gamma}$**:**

For $\boldsymbol{\alpha}$:

$$\pi(\boldsymbol{\alpha}|\boldsymbol{y}) \propto exp\left\{-\frac{1}{2}(\boldsymbol{\alpha} - \widehat{\boldsymbol{\mu}}_{\boldsymbol{\alpha}})'\hat{\Sigma}_{\boldsymbol{\alpha}}^{-1}(\boldsymbol{\alpha} - \widehat{\boldsymbol{\mu}}_{\boldsymbol{\alpha}})\right\}$$

$$\propto N(\widehat{\boldsymbol{\mu}}_{\boldsymbol{\alpha}}, \hat{\Sigma}_{\boldsymbol{\alpha}})I_{(\boldsymbol{\alpha}>0)}$$

For $\boldsymbol{\beta}$:

$$\pi(\boldsymbol{\beta}|\boldsymbol{y}) \propto exp\left\{-\frac{1}{2}(\boldsymbol{\beta} - \widehat{\boldsymbol{\mu}}_{\boldsymbol{\beta}})'\hat{\Sigma}_{\boldsymbol{\beta}}^{-1}(\boldsymbol{\beta} - \widehat{\boldsymbol{\mu}}_{\boldsymbol{\beta}})\right\}$$

$$\propto N(\widehat{\boldsymbol{\mu}}_{\boldsymbol{\beta}}, \hat{\Sigma}_{\boldsymbol{\beta}})I_{(\boldsymbol{\beta}\geq 0)}$$

For $\boldsymbol{\gamma}$:

$$\pi(\boldsymbol{\gamma}|\boldsymbol{y}) \propto exp\left\{-\frac{1}{2}(\boldsymbol{\gamma} - \widehat{\boldsymbol{\mu}}_{\boldsymbol{\gamma}})'\hat{\Sigma}_{\boldsymbol{\gamma}}^{-1}(\boldsymbol{\gamma} - \widehat{\boldsymbol{\mu}}_{\boldsymbol{\gamma}})\right\}$$

$$\propto N(\widehat{\boldsymbol{\mu}}_{\boldsymbol{\gamma}}, \hat{\Sigma}_{\boldsymbol{\gamma}})I_{(|\boldsymbol{\gamma}|>0)}$$

The prior density of $\boldsymbol{\omega}$ is given by

$$f(\boldsymbol{\omega}|\upsilon) \propto \prod_{t=1}^{T} \omega_t^{-\frac{\upsilon}{2}-1} \, exp\left\{\frac{-\upsilon}{2\omega_t}\right\}$$

and its likelihood function is given by

$$L(\boldsymbol{\omega}|\boldsymbol{y}) \propto \prod_{t=1}^{T} \omega_t^{-\frac{1}{2}} \, exp\left\{-\frac{1}{2}\left(\frac{y_t^2}{\omega_t \sigma h_t}\right)\right\}.$$

Hence, by Bayes' theorem, the joint posterior density of $\boldsymbol{\omega}$ is given by

$$\pi(\boldsymbol{\omega}|\boldsymbol{y}) \propto \prod_{t=1}^{T} \omega_t^{-\frac{\upsilon+1}{2}-1} \, exp\left\{\frac{-\tilde{\upsilon}_t}{2\omega_t}\right\}$$

which is the kernel of an inverse gamma density with parameters $\frac{\upsilon+1}{2}$ and $\tilde{\upsilon}_t = \frac{y_t^2}{\sigma h_t} + \upsilon$.

Following Deschamps' (2006) choice of the prior density of the degrees of freedom parameter υ, the translated exponential density with parameters $\mu > 0$ and $\lambda \geq 0$ and is given by

$$f(\upsilon) = \mu e^{-\mu(\upsilon-\lambda)}$$

is used. The joint density of the vector $\boldsymbol{\omega} = (\omega_1, \dots, \omega_T)'$ conditional on υ is

$$f(\boldsymbol{\omega}|\upsilon) = \left(\frac{\upsilon}{2}\right)^{\frac{T\upsilon}{2}} \left[\Gamma\left(\frac{\upsilon}{2}\right)\right]^{-T} \prod_{t=1}^{T} \omega_t^{-\frac{\upsilon}{2}-1} \, exp\left\{\frac{-\upsilon}{2\omega_t}\right\}.$$

Thus, the posterior (proposal) density of υ is

$$\pi(\upsilon|\boldsymbol{\omega}) \propto \left(\frac{\upsilon}{2}\right)^{\frac{T\upsilon}{2}} \left[\Gamma\left(\frac{\upsilon}{2}\right)\right]^{-T} exp\{-\varphi\upsilon\}$$

where $\varphi = \frac{1}{2}\left[\sum_{t=1}^{T}\left(\frac{1}{\omega_t} + \ln \omega_t\right)\right] + \mu$.

APPLICATION TO REAL DATA

TABLE I: Descriptive Statistics of the JFC Return Series

	Statistic	p-value
Sample Size	1692	
Minimum	-10.48796	
Maximum	9.40701	
Mean	0.05819	
Standard Deviation	1.81376	
Skewness	0.00972	
Kurtosis	2.84917	
Jarque-Bera Test for Normality	577.75	$< 2.2 \times 10^{-16}$*
Augmented Dickey-Fuller Test for Stationarity	-13.285	0.01*
Box-Pierce Test for Serial Correlation	34.083	5.282×10^{-9}*
Ljung-Box Test for Heteroscedasticity	34.143	5.121×10^{-9}*

- * on the p-value implies rejection of the null hypothesis H_0.
- Augmented Dickey-Fuller Test H_0: The series is not stationary.
- Jarque-Bera Test H_0: The series is normally distributed.
- Box-Pierce Test H_0: The series has no serial correlation.
- Ljung-Box Test H_0: There is no ARCH effect present in the series.

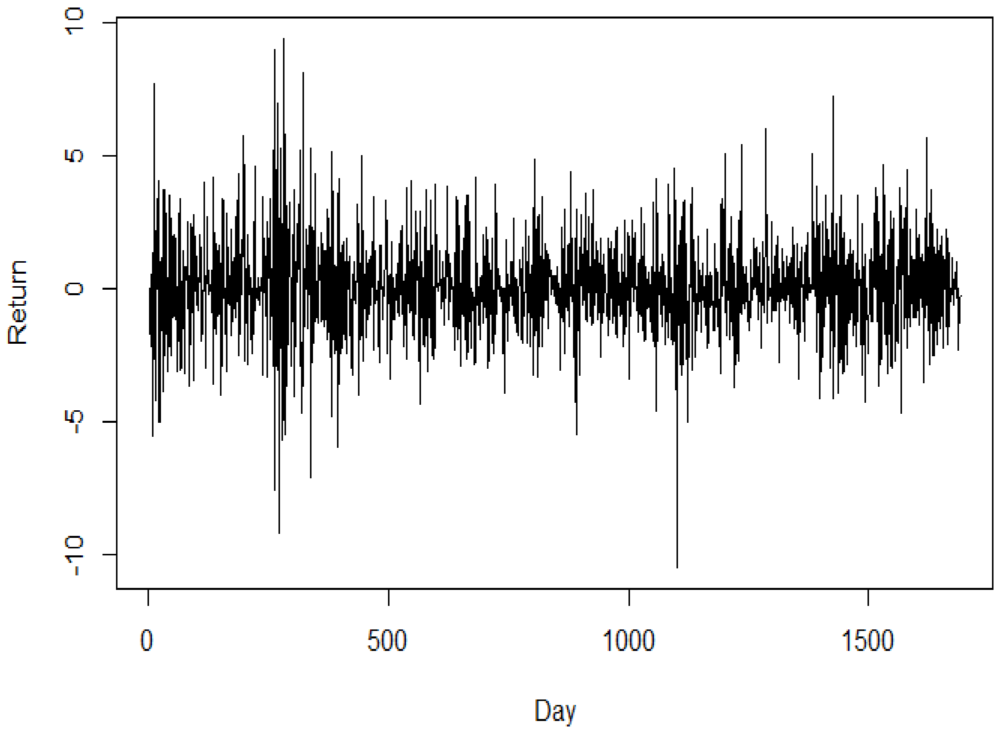

Fig. 1. Historical Plot of the JFC Return Series

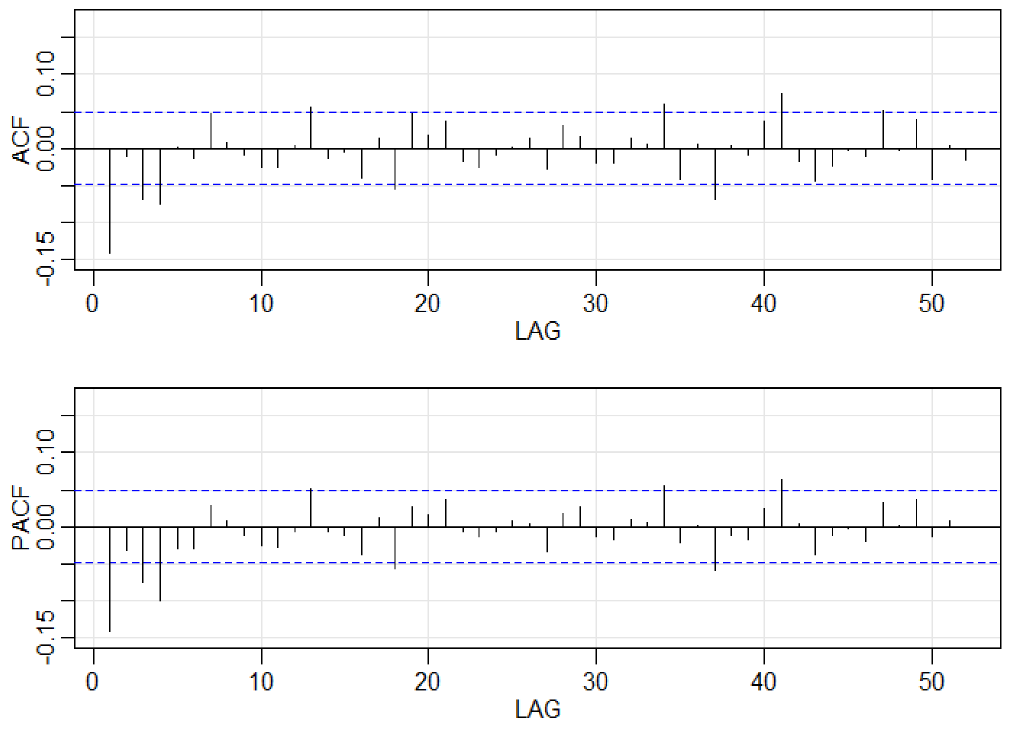

Fig. 2. Correlogram of the ACF and PACF of JFC Return Series

Selection of ARIMA(*p*,*d*,*q*) Model

TABLE II: Comparison of the Candidate Mean Models

Model	AIC
ARMA(1,0)	6787.09
ARMA(0,1)	6784.51
ARMA(1,1)	6770.72

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Fig. 3. Residual Plot

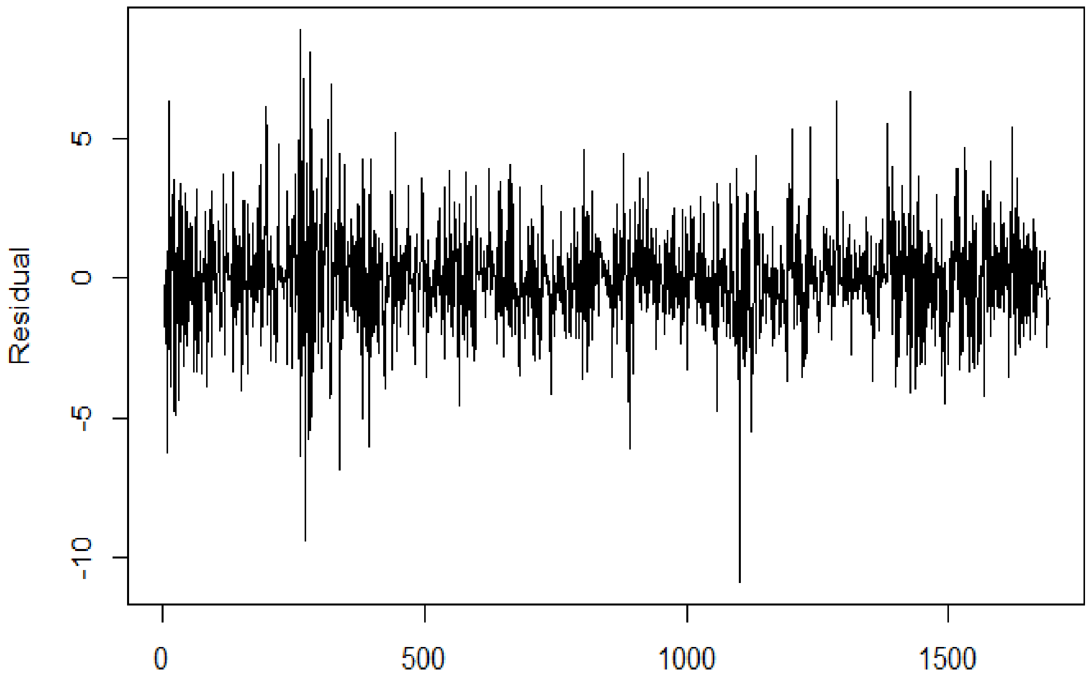

TABLE III: Diagnostics in the Mean Model Residuals

	Statistic	p-value
White Noise Test	15.883	0.02621*
Ljung-Box Test for Heteroscedasticity	57.335	3.675×10^{-14}*
Jarque-Bera Test for Normality	537.88	$< 2.2 \times 10^{-16}$*

- * on the p-value implies rejection of the null hypothesis H_0.
- Ljung-Box test is performed on the squared residuals of the mean model.
- White Noise Test H_0: The residuals follow a white noise behavior.
- Ljung-Box Test H_0: There is no ARCH effect present in the series.
- Jarque-Bera Test H_0: The series is normally distributed.

Selection of A-PARCH(p,d,q) Model

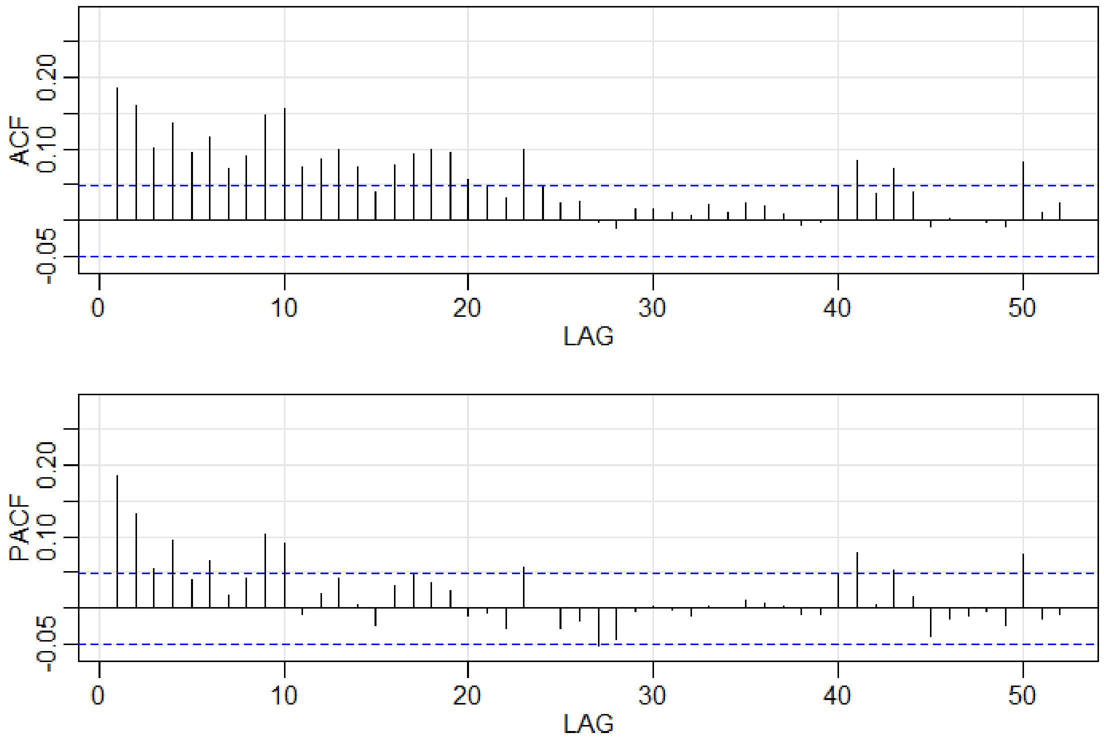

Fig. 4. Correlogram of the ACF and PACF of Squared Residuals

TABLE IV: Bayesian Estimates of the ARMA(1,1) – A-PARCH(1,1) with Student's *t*-distributed Innovations Parameters

Parameter	Estimates
ϕ_1	0.29589
θ_1	-0.46822
α_0	0.18483
α_1	0.18401
γ_1	0.31214
β_1	0.74935
δ	0.79492
ν	5.01306

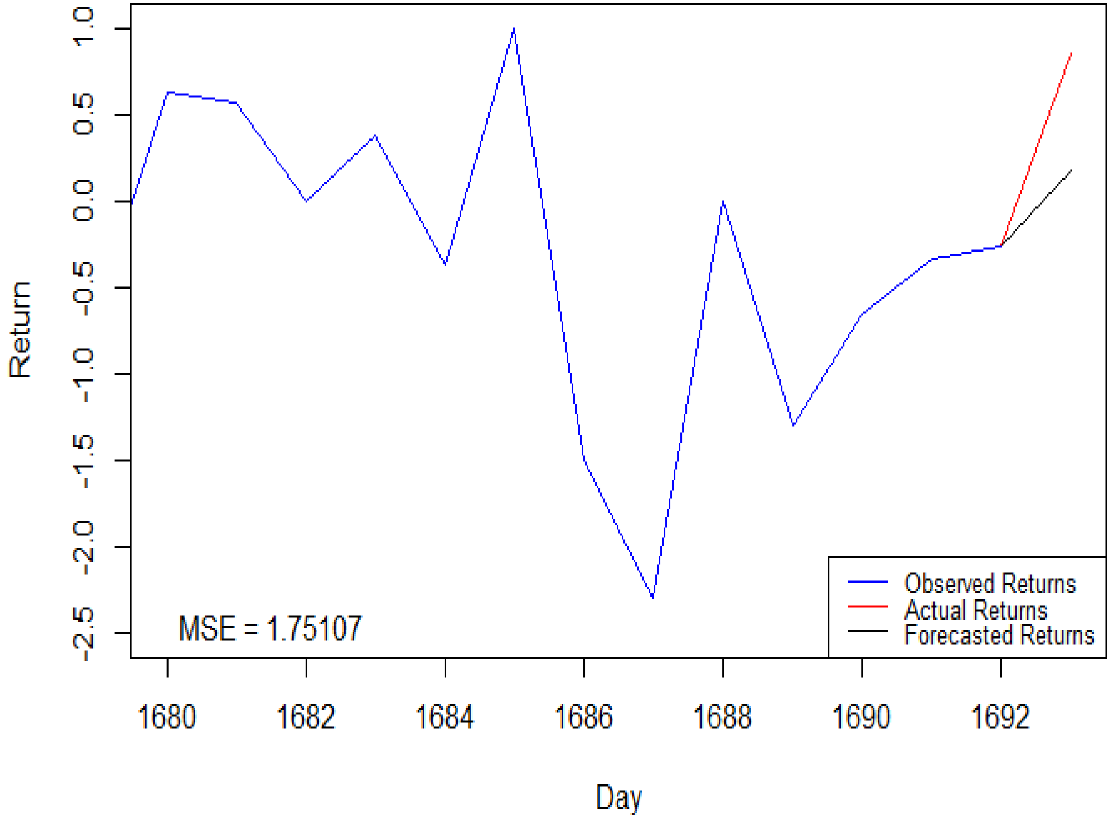

Fig. 5. One-Step Ahead Forecast of JFC Returns

Thank you