Forecasting Extreme Economic Misery Indices

Peter Julian A Cayton

School of Statistics, University of the Philippines

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Introduction

Economic Misery: the combination of misery due to the inflation of prices and the misery of joblessness as measured by the unemployment rate. (Okun 1971)

The idea of monitoring economic misery in the Philippine context is broken to individual components. The executive branch of the national government conducts labor policies to facilitate the improvement of employment while the central bank devises and executes monetary policies which influences consumer price movements, among others.

Introduction

A gap in insights for scenarios of extreme economic misery also exists, especially that effects of extreme economic scenarios tend to have impacts that can persists long after the occurrence of the extreme event.

The research proposes a methodology of monitoring and forecasting the extreme values of the misery index and its components using financial risk management statistics. The proposed methodologies are based on the Value-at-Risk [VaR] approach but are estimated using parametric and semiparametric statistics.

Introduction

Objectives:

(1) to propose a methodology of estimating extreme levels of economic misery through the VaR approach, the extreme level being called Misery-at-Risk [MaR]; and

(2) to evaluate the utility of the methodology through hold-out analysis, in which more recent data are held out from estimation as checks on the approach and analyze recent economic events.

Review of Related Literature
Misery Indices

Okun's Misery Index (Okun 1971):

$$M = \pi + u_1$$

Job Misery Index (Mapa, et al. 2013; Beja 2014; Mapa, et al. 2015):

$$m_{job} = u_1 + u_2$$

$$u_2 = u_{2,unadj}\left(1 - \frac{u_1}{100}\right)$$

Modified Misery Index;

$$M_{prop} = \pi + m_{job} = \pi + u_1 + u_2$$

Review of Related Literature

Value-at-Risk

Returns data (r_t): If $\{P_t\}_{t=1}^{T}$ is the price of a non-dividend paying financial instrument,

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Value-at-risk VaR_α: a measure of market risk defined as the loss that can occur given a coverage probability α that the loss will not be exceeded (Jorion 2007).

$$P(r_t < VaR_\alpha) = 1 - \alpha$$
$$VaR_\alpha = F_{r_t}^{-1}(1 - \alpha)$$

Review of Related Literature
Box-Jenkins Methodology

Auto-Regressive Integrated Moving Average [ARIMA] (Box, Jenkins, & Reinsel 2008):

$$y_t \sim SARIMA(p,d,q) \times (P,D,Q)_s$$

$$\Phi_{\mathrm{P}}(B^s)\phi_p(B)(1-B^s)^D(1-B)^d y_t = \delta + \Theta_{\mathrm{Q}}(B^s)\theta_q(B)\epsilon_t, \qquad \epsilon_t \sim WN(0,\sigma^2)$$

Backshift Operator: $B^i y_t = y_{t-i}$

Intercept δ: This parameter may be set to zero in some instances. When a model has $d+D=1$, it is often called the drift parameter.

Review of Related Literature
Box-Jenkins Methodology

White Noise ϵ_t: zero mean and constant variance and covariance between the variables in the process is zero. It is often modeled as a Gaussian or normal distribution for the purpose of specifying a likelihood function for estimating ARIMA parameters. The density function f_Z of the normal distribution is:

$$f_Z(z;\ \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}(z-\mu)^2\right\}, \qquad z \in \mathbb{R}$$

Review of Related Literature
Box-Jenkins Methodology

Simple differencing of order d is described by $(1 - B)^d$, which is also called integrated model of order d, or $I(d)$.

Seasonal differencing of order D for s seasons in time series is described by $(1 - B^s)^D$, or a seasonally integrated model of order D and season s and is denoted as $SI(D)_s$.

Review of Related Literature
Box-Jenkins Methodology

$\phi_p(B)$: Auto-Regressive [AR] model of order p, or $AR(p)$

$$\phi_p(B) = 1 - \sum_{i=1}^{p} \phi_i B^i$$

$\Phi_{\mathrm{P}}(B^s)$: Seasonal AR models of order P with s seasons, $SAR(P)_s$

$$\Phi_P(B^s) = 1 - \sum_{i=1}^{P} \Phi_i B^{s \times i}$$

Review of Related Literature
Box-Jenkins Methodology

$\theta_q(B)$: Moving Average Model of order q, $MA(q)$

$$\theta_q(B) = 1 + \sum_{i=1}^{q} \theta_i B^i$$

$\Theta_Q(B^s)$: Seasonal moving average models of order Q and s seasons, $SMA(Q)_s$

$$\Theta_Q(B^s) = 1 + \sum_{i=1}^{q} \Theta_i B^{s \times i}$$

Review of Related Literature
Box-Jenkins Methodology

Maximum Likelihood Estimation: $z_t = (1 - B^s)^D (1 - B)^d y_t$

$$L\left(\delta, \phi_1, \ldots, \phi_p, \theta_1, \ldots, \theta_q, \Phi_1, \ldots, \Phi_P, \Theta_1, \ldots \Theta_Q \middle| z_1, \ldots, z_T\right) = \prod_{i=1}^{T} f_Z(z_t; \mu = \mu_{ARMA}, \sigma^2)$$

Review of Related Literature
Box-Jenkins Methodology

Automatic ARIMA Modeling and Forecasting (Hyndman & Khandakar 2008):

1. Differencing: seasonal differencing is determined first by whether $D = 0$ or $D = 1$ by the Canova-Hansen test (Canova & Hansen 1995). Then, nonseasonal differencing order d is determined by repeated use of the KPSS test (Kwiatkowski, et al. 1992) until the first instance of stationarity

2. Intercept: If $d + D \leq 1$, then $\delta \neq 0$, while for all other cases, $\delta = 0$

Review of Related Literature
Box-Jenkins Methodology

Automatic ARIMA Modeling and Forecasting (Hyndman & Khandakar 2008):

3. ARMA: the proper SARMA orders are automatically selected by which minimizes the AIC (Akaike 1974) and where, by default settings, $0 \leq p+q \leq 5$ and $0 \leq P+Q \leq 2$, the models are appropriate stationary and invertible, and no other problems in the estimation process are encountered

$$AIC = \begin{cases} -2\log(L_{\max}) + 2(p+q+P+Q+1) & if\ \delta \neq 0 \\ -2\log(L_{\max}) + 2(p+q+P+Q) & if\ \delta = 0 \end{cases}$$

Review of Related Literature
Bootstrapping

Sieve Bootstrap (Buhlmann 1997)

Suppose that $W = W(z_1, \ldots, z_T)$ is a target statistic computed from a time series dataset $\{z_t\}_{t=1}^{T}$ and let n_B is number of performed resamples.

Step 1: Fit an $AR(\infty)$ model, of which a SARIMA model is a restricted form, on $\{z_t\}_{t=1}^{T}$ and extract the residuals $\{\hat{\epsilon}_t\}_{t=1}^{T}$ and fitted values $\{\hat{z}_t\}_{t=1}^{T}$.

Review of Related Literature
Bootstrapping

Sieve Bootstrap (Buhlmann 1997)

Step 2: Resampling Procedure; for each i^{th} instance, $i = 1,2, \ldots, n_B$,

Sub-step 1: Generate a random sample $\left\{\hat{\epsilon}_t^{(i)}\right\}_{t=1}^{T}$ from $\{\hat{\epsilon}_t\}_{t=1}^{T}$

Sub-step 2: Evaluate $\left\{z_t^{(i)} = \hat{z}_t + \hat{\epsilon}_t^{(i)}\right\}_{t=1}^{T}$

Sub-step 3: Evaluate $W^{(i)} = W\left(z_1^{(i)}, \ldots, z_T^{(i)}\right)$

Review of Related Literature
Bootstrapping

Sieve Bootstrap (Buhlmann 1997)

Step 3: Let $I(A) = 1$ if statement A is true, and $I(A) = 0$ otherwise. The estimator for the sampling distribution F_W of the target statistic is

$$\hat{F}_W(w) = \frac{1}{n_B}\sum_{i=1}^{n_B} I\left(W^{(i)} \leq w\right)$$

Review of Related Literature
Optimal Forecast Reconciliation

Hyndman, et al. (2011) proposed an optimal forecast reconciliation approach in combining forecasts.

$\hat{Y}_t = \left[\widehat{M}_{prop,t}, \widehat{m}_{job,t}, \hat{p}_t, \hat{u}_{1,t}, \hat{u}_{2,t}\right]'$ be the unreconciled forecasts for overall misery, job misery, and the individual components at time t, respectively;

$\hat{Y}_t^* = \left[\hat{p}_t^*, \hat{u}_{1,t}^*, \hat{u}_{2,t}^*\right]'$ are the target but unknown reconciled individual time series forecasts that generates the forecasts for the aggregates.

Review of Related Literature
Optimal Forecast Reconciliation

The summing matrix S:

$$S = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Review of Related Literature
Optimal Forecast Reconciliation

$\hat{Y}_t$, S , and $\hat{Y}_t^*$ are linked in a regression equation below with $e_t = \left[e_{1,t}, e_{2,t}, e_{3,t}, e_{4,t}, e_{5,t}\right]' \sim N_5(0, \Sigma_t)$, where Σ_t is the covariance matrix of the unreconciled forecasts:

$$\hat{Y}_t = S\hat{Y}_t^* + e_t$$

By ordinary least squares approach, which assumes that Σ_t is a diagonal matrix of similar values, the estimated reconciled forecasts $\tilde{Y}_t^*$ for each time t are:

$$\tilde{Y}_t^* = (S'S)^{-1}S'\hat{Y}_t$$

Proposed Measure
Misery-at-Risk

Misery-at-risk MaR_{α}: economic misery that can occur given a risk probability α that the misery will not be exceeded.

$MaR_{0.90}$ = 90% misery-at-risk; 90% probability that misery will be less than or equal to such value.

The overall, job, inflation, unemployment, and underemployment MaRs with forecast horizon h are denoted as $MaR^{M}_{\alpha,T+h}$, $MaR^{JM}_{\alpha,T+h}$, $MaR^{P}_{\alpha,T+h}$, $MaR^{U_1}_{\alpha,T+h}$, and $MaR^{U_2}_{\alpha,T+h}$, respectively

Proposed Measure
Misery-at-Risk

The procedure for generating the MaR is discussed below:

Step 1: Misery Calculation. Solve for the job misery and overall misery indices.

For the paper, the misery is solved per quarter, as unemployment and underemployment are quarterly indices. To solve for headline inflation, the monthly consumer price index [CPI] is first averaged per quarter and year-on-year growth rates are solved from the converted quarterly CPI.

Proposed Measure
Misery-at-Risk

The procedure for generating the MaR is discussed below:

Step 2: SARIMA Modeling. Using auto.arima(), fit SARIMA models on overall misery $\{M_t\}_{t=1}^{T}$, job misery $\{m_{job,t}\}_{t=1}^{T}$, inflation $\{\pi_t\}_{t=1}^{T}$, unemployment $\{u_{1,t}\}_{t=1}^{T}$, and adjusted underemployment $\{u_{2,t}\}_{t=1}^{T}$. Extract the residuals $\{\hat{\epsilon}_{M,t}\}_{t=1}^{T}$, $\{\hat{\epsilon}_{JM,t}\}_{t=1}^{T}$, $\{\hat{\epsilon}_{\pi,t}\}_{t=1}^{T}$, $\{\hat{\epsilon}_{u_1,t}\}_{t=1}^{T}$, and $\{\hat{\epsilon}_{u_2,t}\}_{t=1}^{T}$; and fitted values $\{\hat{z}_{M,t}\}_{t=1}^{T}$, $\{\hat{z}_{JM,t}\}_{t=1}^{T}$, $\{\hat{z}_{\pi,t}\}_{t=1}^{T}$, $\{\hat{z}_{u_1,t}\}_{t=1}^{T}$ and $\{\hat{z}_{u_2,t}\}_{t=1}^{T}$.

Proposed Measure
Misery-at-Risk

The procedure for generating the MaR is discussed below:

Step 3: Bootstrapping and Forecast Reconciliation. For each i^{th} instance, $i = 1,2,\dots,n_B$:

- Sub-step 1: Resampling. Generate a random sample $\left\{\hat{\epsilon}_{K,t}^{(i)}\right\}_{t=1}^{T}$ from $\left\{\hat{\epsilon}_{K,t}\right\}_{t=1}^{T}$ for each $K \in \{M, JM, \pi, u_1, u_2\}$.

- Sub-step 2: Reproduction. Evaluate $\left\{z_{K,t}^{(i)} = \hat{z}_{K,t} + \hat{\epsilon}_{K,t}^{(i)}\right\}_{t=1}^{T}$ for each index K.

Proposed Measure
Misery-at-Risk

The procedure for generating the MaR is discussed below:

Step 3: Bootstrapping and Forecast Reconciliation. For each i^{th} instance, $i = 1,2, \dots, n_B$:

- Sub-step 3: Forecasting. Evaluate the forecast function $\hat{Y}_{K,T+h}^{(i)} = \hat{Y}\left(z_{K,1}^{(i)}, \dots, z_{K,T}^{(i)}\right)$ as forecasts based on the SARIMA model estimated in Step 2 with forecast horizon h for each index K.

Proposed Measure
Misery-at-Risk

The procedure for generating the MaR is discussed below:

Step 3: Bootstrapping and Forecast Reconciliation. For each i^{th} instance, $i = 1,2,\ldots,n_B$:

- Sub-step 4: Reconciliation. $\hat{Y}_{T+h}^{(i)} = \left[\hat{Y}_{M,T+h}^{(i)}, \hat{Y}_{JM,T+h}^{(i)}, \hat{Y}_{\pi,T+h}^{(i)}, \hat{Y}_{u_1,T+h}^{(i)}, \hat{Y}_{u_2,T+h}^{(i)}\right]'$ as a vector of unreconciled forecasts. The reconciled forecasts $\tilde{Y}_{T+h}^{(i)} = \left[\tilde{Y}_{\pi,T+h}^{(i)}, \tilde{Y}_{u_1,T+h}^{(i)}, \tilde{Y}_{u_2,T+h}^{(i)}\right]'$, $\tilde{Y}_{JM,T+h}^{(i)}$, and $\tilde{Y}_{M,T+h}^{(i)}$ are solved by:
- $\tilde{Y}_{T+h}^{(i)} = (S'S)^{-1}S'\hat{Y}_{T+h}^{(i)};\ \tilde{Y}_{JM,T+h}^{(i)} = \tilde{Y}_{u_1,T+h}^{(i)} + \tilde{Y}_{u_2,T+h}^{(i)};\ \tilde{Y}_{M,T+h}^{(i)} = \tilde{Y}_{\pi,T+h}^{(i)} + \tilde{Y}_{JM,T+h}^{(i)}.$

Proposed Measure
Misery-at-Risk

The procedure for generating the MaR is discussed below:

Step 4: Forecast Distribution Estimation. The estimated forecast distribution is for misery index I and horizon h is $\hat{F}_{\tilde{Y}_{K,T+h}}$ and is solved by

$$\hat{F}_{\tilde{Y}_{K,T+h}}(w) = \frac{1}{n_B} \sum_{i=1}^{n_B} I\left(\tilde{Y}_{K,T+h}^{(i)} \leq w\right).$$

Proposed Measure
Misery-at-Risk

The procedure for generating the MaR is discussed below:

Step 5: Misery-at-Risk Estimation. Evaluate $MaR_{\alpha,T+h}^{K}$ for each K by:

$$MaR_{\alpha,T+h}^{K} = \hat{F}_{\tilde{Y}_{K,T+h}}^{-1}(1-\alpha) = \tilde{Y}_{K,T+h}^{[(1-\alpha)n_B]}.$$

The notation $\tilde{Y}_{K,T+h}^{[(1-\alpha)n_B]}$ means that the $(1-\alpha)n_B$-th smallest value from the forecast distribution will be used, with $(1-\alpha)n_B$ truncated to the nearest integer.

Proposed Measure
Real Data Application

The proposed measure will be applied to the Philippine inflation, unemployment, and underemployment data from Q1 1995 to Q2 2019 as made available by the Philippine Statistics Authority.

Inflation is the headline inflation computed as the year-on-year change in the CPI:

$$\pi_t = \frac{CPI_t - CPI_{t-4}}{CPI_{t-4}} \times 100\%$$

Proposed Measure
Real Data Application

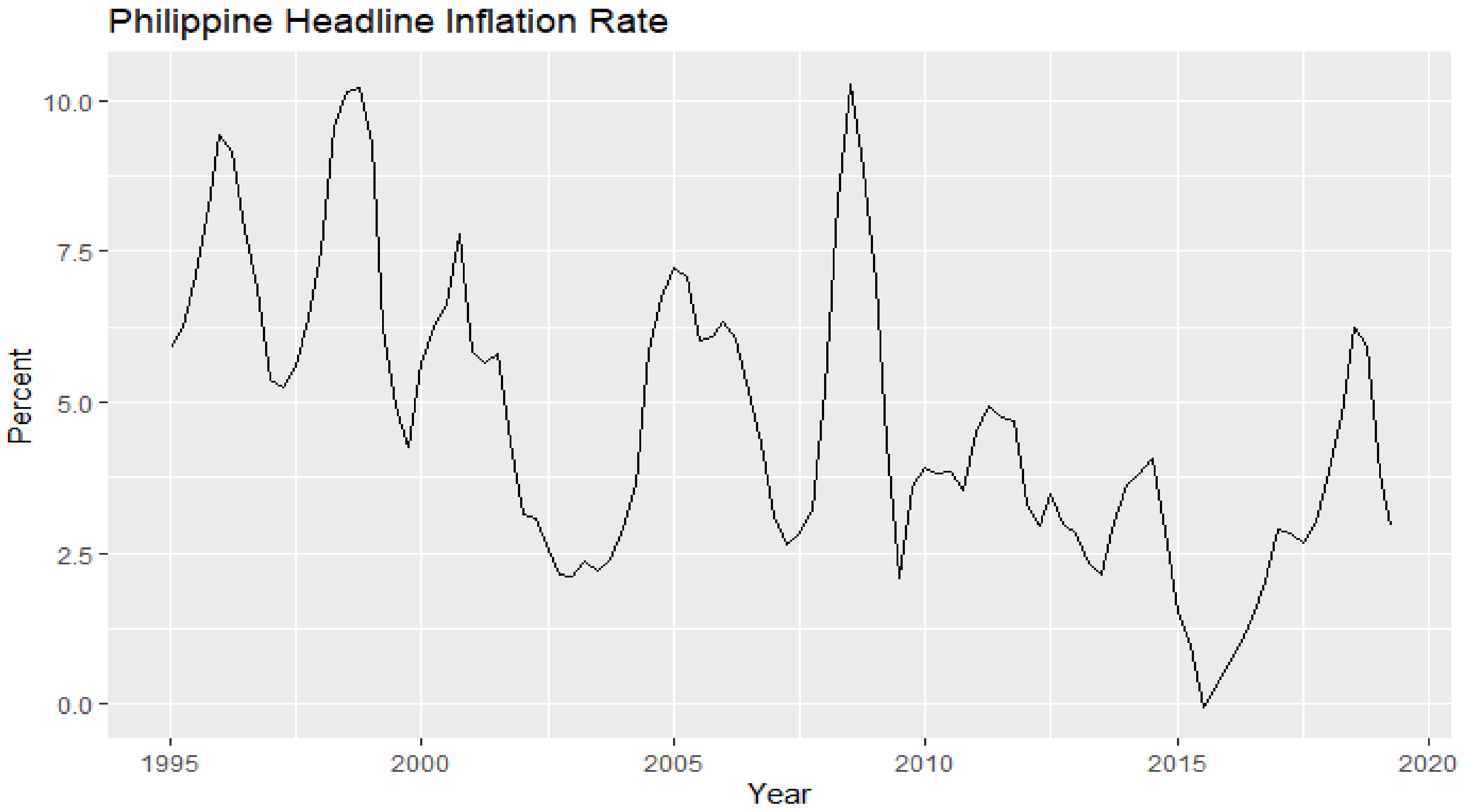
Since CPI is reported monthly, a quarterly average is solved to derive the quarterly CPI values. As the CPI changed base years in 2018 from 2006=100 to 2012=100, the transform for data points on or before 2011 are shown below, in compliance with symmetric time and circularity properties of price indices (Diewert 1993):

$$CPI_t^{(2006=100 \Rightarrow 2012=100)} = \frac{CPI_t^{(2006=100)}}{\frac{1}{4}\sum_{t=Q1\,2012}^{Q4\,2012} CPI_t^{(2006=100)}} \times 100.$$

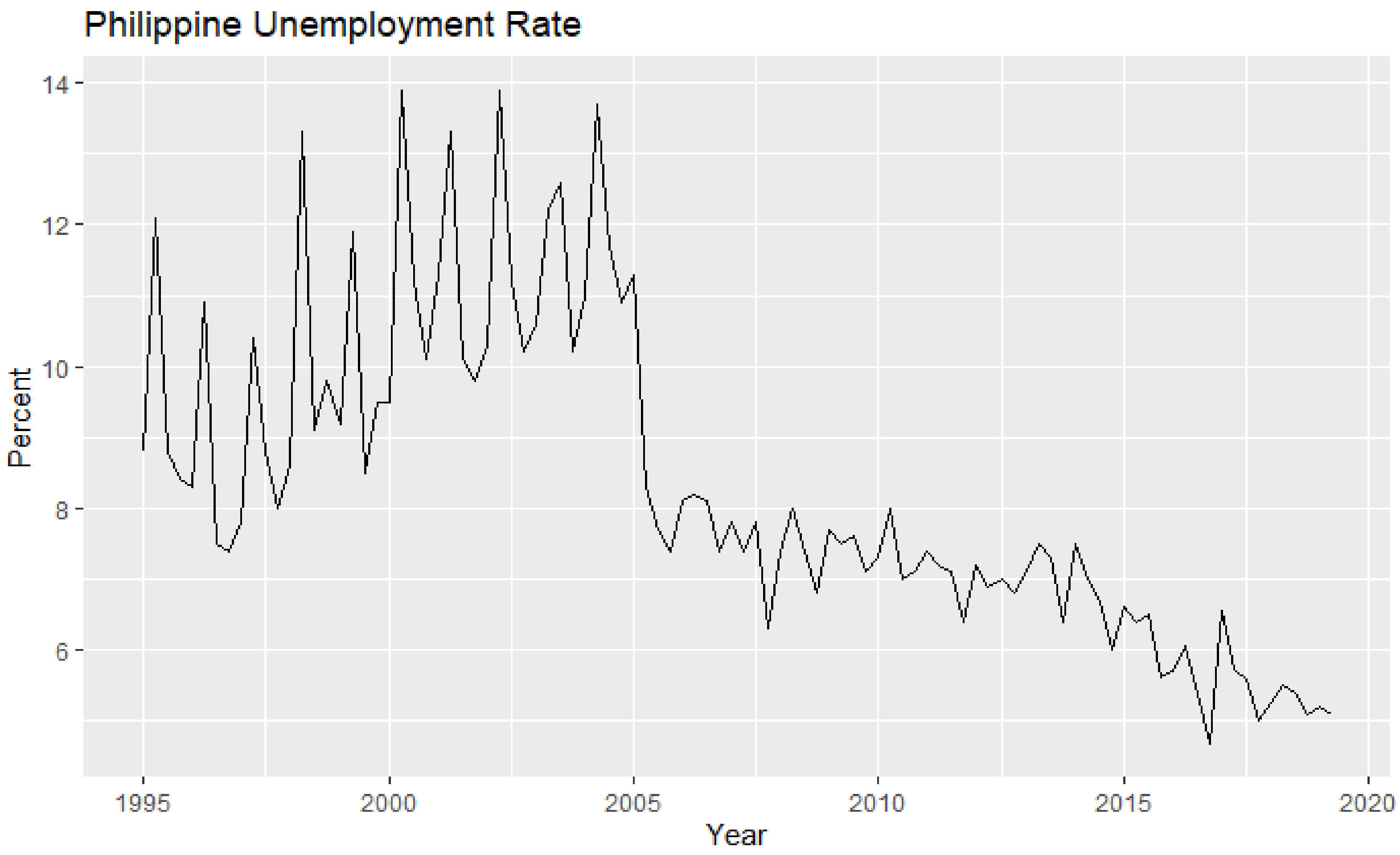
Proposed Measure
Real Data Application

The MaR is derived with two coverage probabilities: 80% and 90%. MaR at 80% can be used as an early warning to preempt worsening misery while the exceedance of the 90% MaR would be indicative of severe misery.

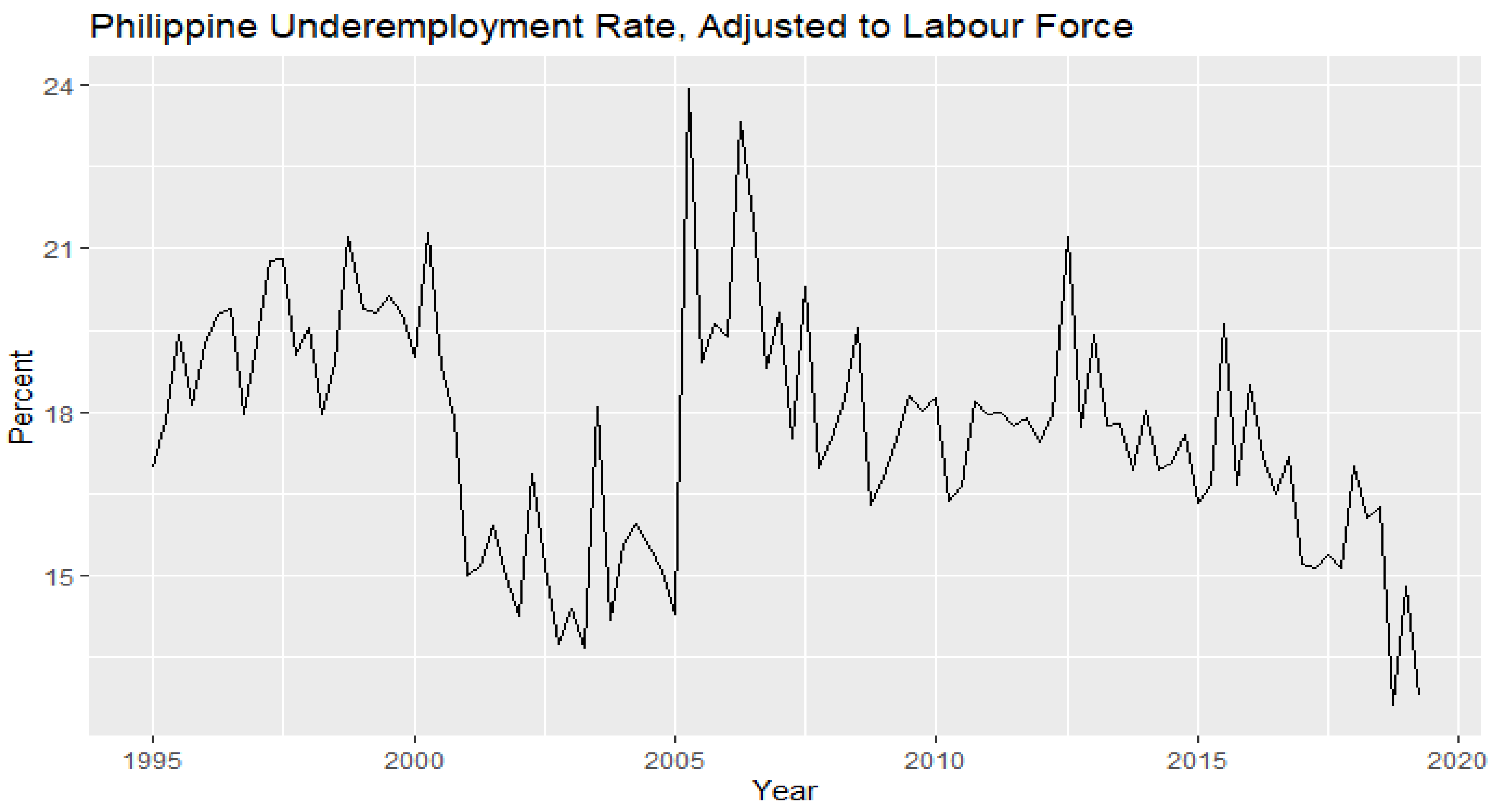
Results and Discussion
Descriptive Analyses of Indices



Results and Discussion
Descriptive Analyses of Indices



Results and Discussion
Descriptive Analyses of Indices



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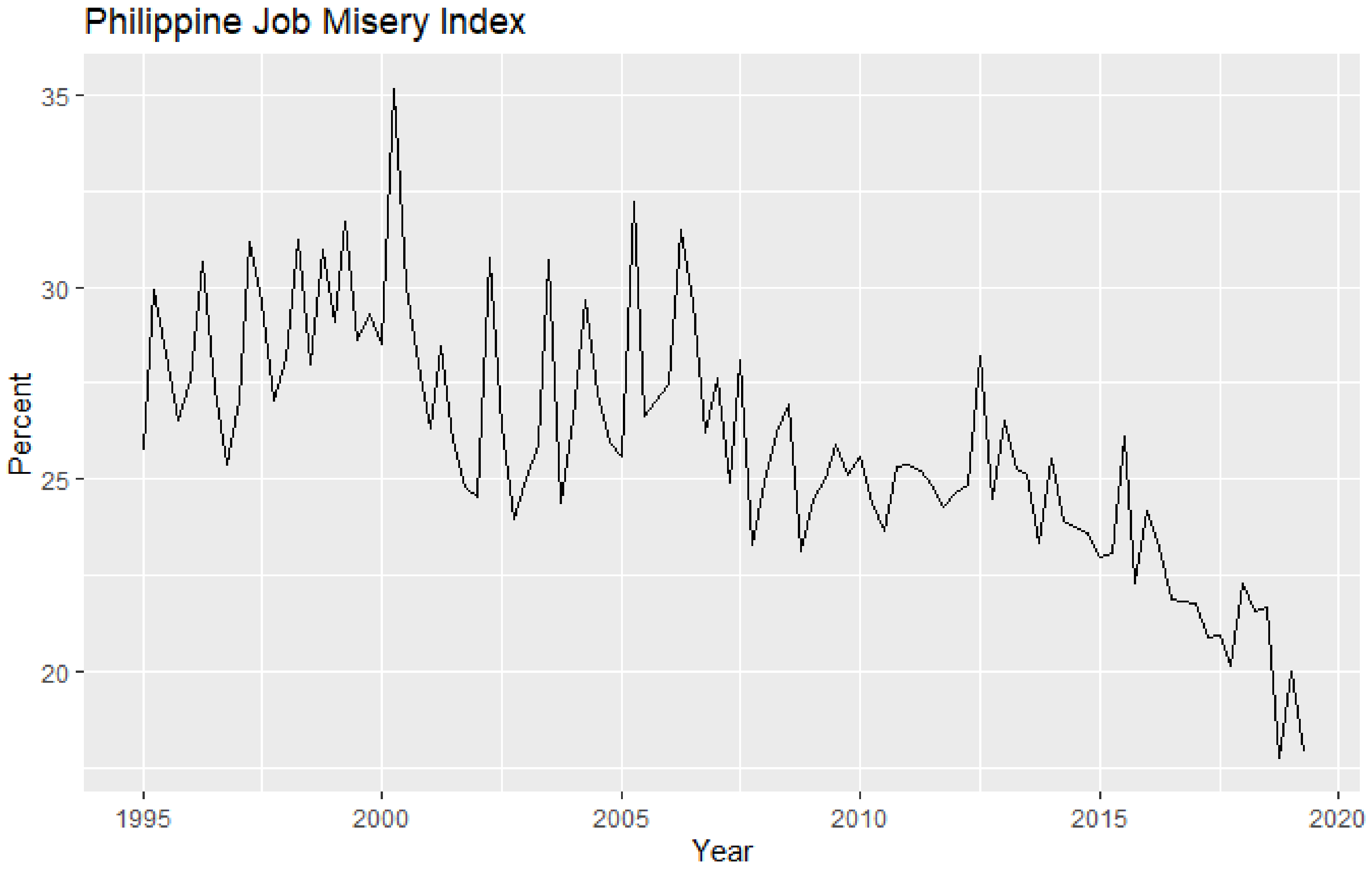
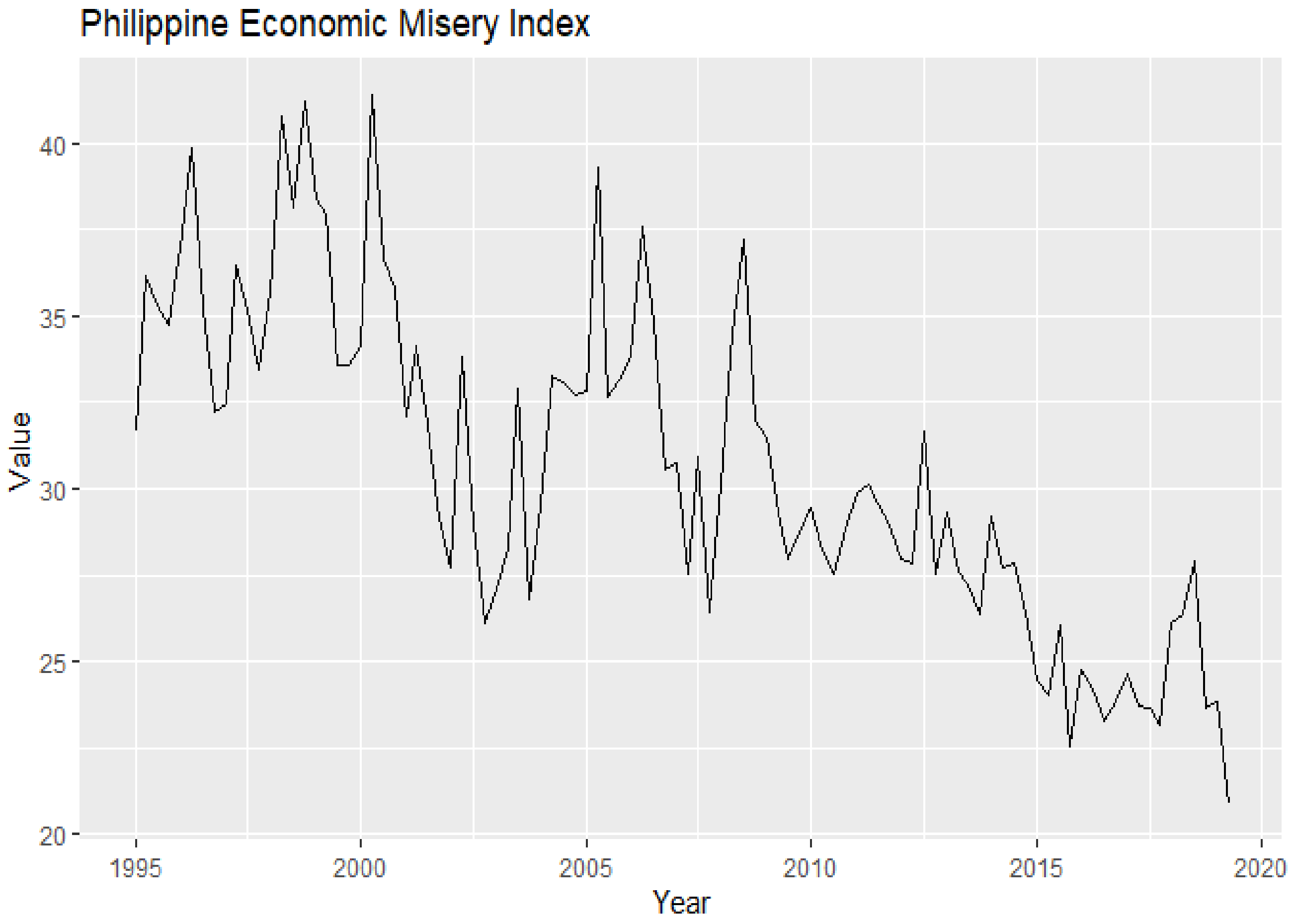
Descriptive Analyses of Indices



Results and Discussion
Descriptive Analyses of Indices



Results and Discussion
Out-of-Sample Performance

Q1 1995 to Q4 2016 are used as training data periods while the rest is used as test data for forecast performance assessment.

Model Parameters	Economic Misery	Inflation	Job Misery	Unemployment	Underemployment
d	1	1	1	0	0
D	0	0	0	1	0
$\hat{\delta}$	--	-0.0610	--	--	17.8858
$se(\hat{\delta})$	--	0.0306	--	--	0.5490
Nonseasonal Terms					
$\hat{\phi}_1$	0.6391	--	--	0.8266	0.8163
$se(\hat{\phi}_1)$	0.1014	--	--	0.0825	0.0990
$\hat{\theta}_1$	-0.9554	0.4530	-0.7101	-0.2786	-0.4841
$se(\hat{\theta}_1)$	0.0376	0.0918	0.0815	0.1285	0.1404
Seasonal Terms					
$\hat{\Phi}_1$	0.9492	--	0.9454	-0.5454	-0.3702
$se(\hat{\Phi}_1)$	0.0486	--	0.0457	0.0900	0.2001
$\hat{\Theta}_1$	-0.7664	-0.7681	-0.7431	--	0.6889
$se(\hat{\Theta}_1)$	0.0952	0.0770	0.0972	--	0.1494
AIC	412.96	198.87	357.57	231.28	340.60

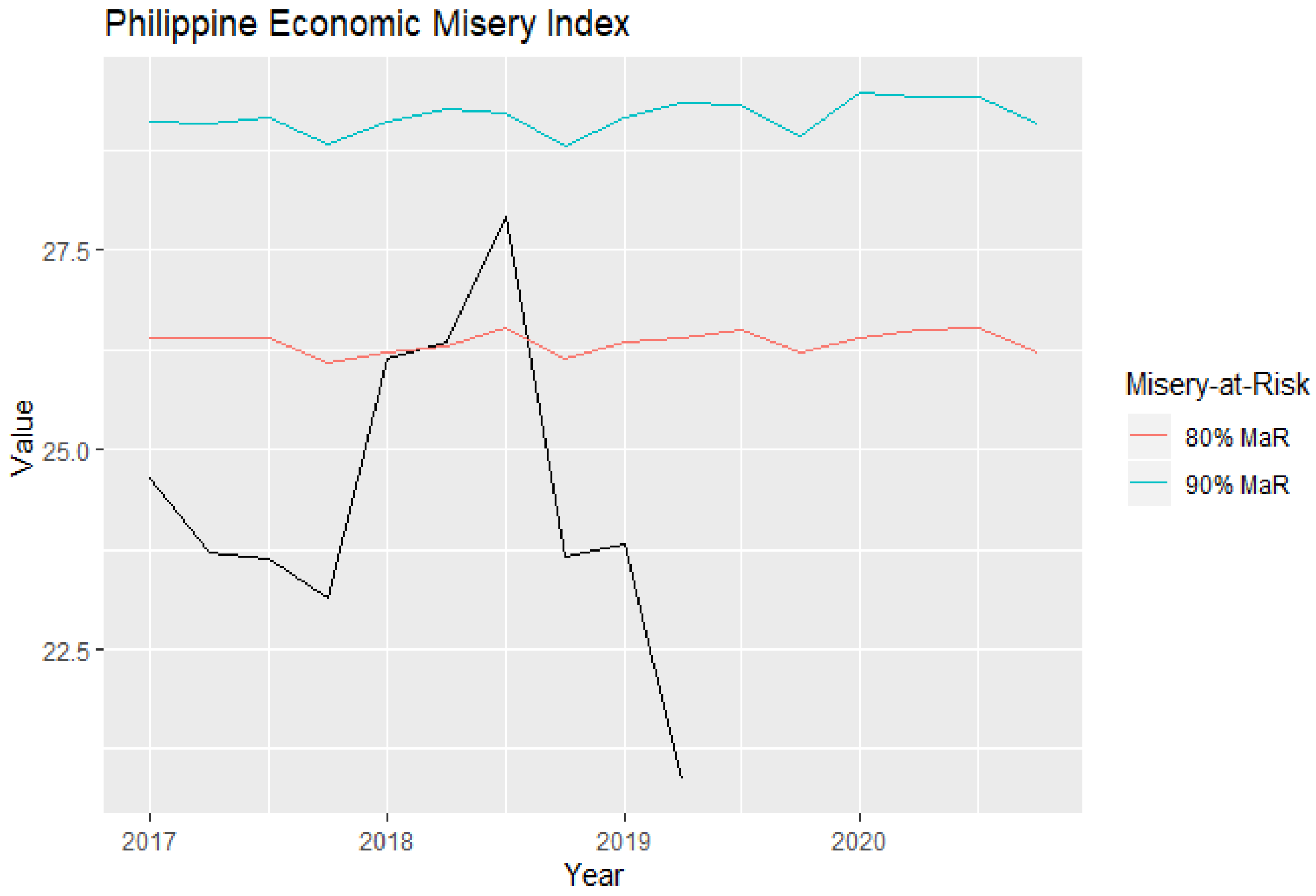
Results and Discussion
Out-of-Sample Performance

	Economic Misery	Inflation	Job Misery	Unemployment	Underemployment
ME	-0.1175	1.8969	-2.0143	0.4343	-2.4487
RMSE	1.7996	2.4076	2.3912	0.5993	2.7855
MAE	1.4762	1.8969	2.0143	0.4351	2.4487
MAPE (in %)	6.0653	42.0931	10.3769	7.7276	17.2429
MPE (in %)	-1.0301	42.0931	-10.3769	7.7137	-17.2429

Results and Discussion
Hold-Out Analysis of Misery-at-Risk

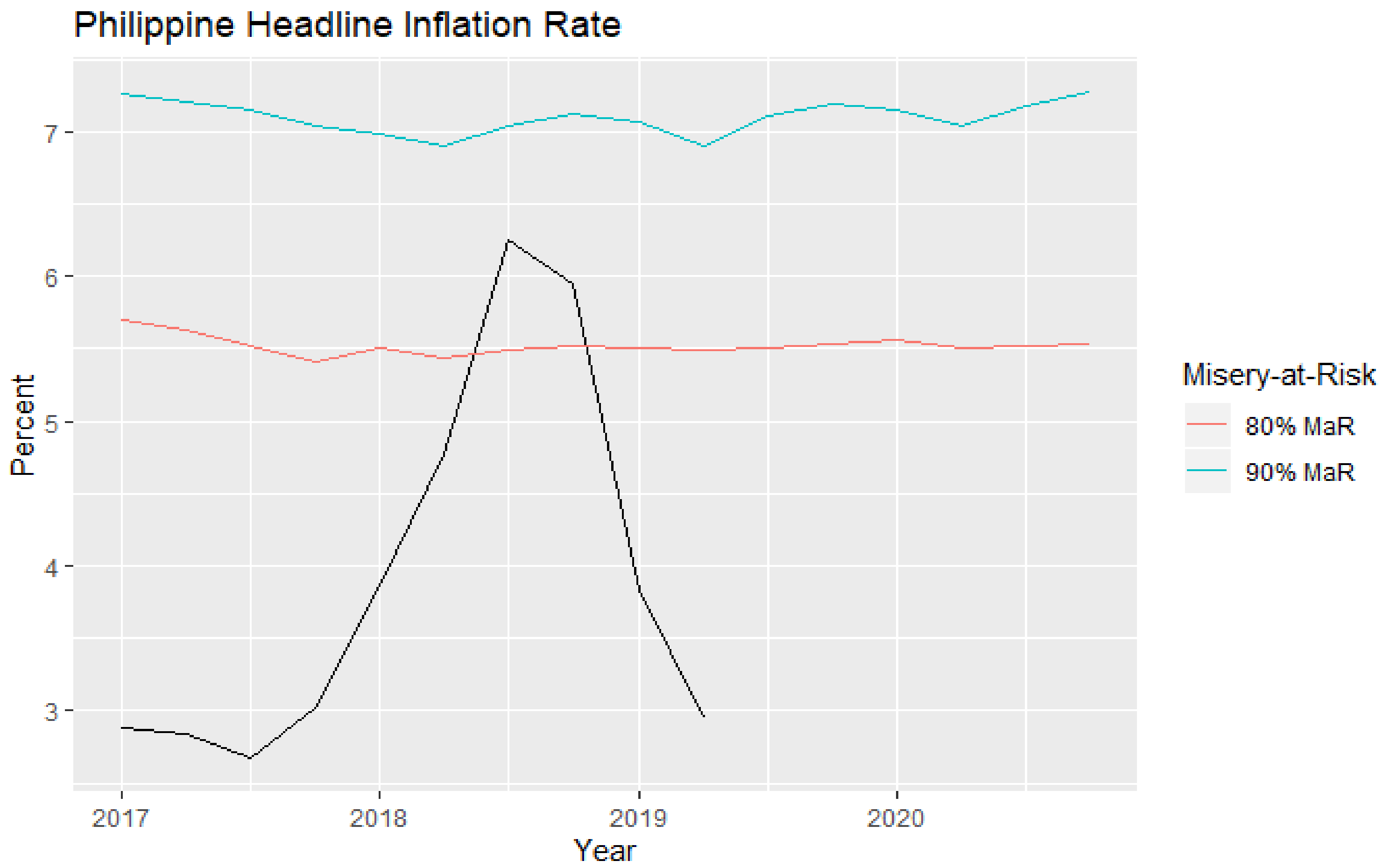
Miseries-at-risk for the indices at 80% and 90% are computed for the test periods 2017 to 2020. The realized index values from 2017 to 2019 Q2 and MaRs are plotted in a line graph.

Results and Discussion
Hold-Out Analysis of Misery-at-Risk



Results and Discussion
Hold-Out Analysis of Misery-at-Risk

Results and Discussion
Hold-Out Analysis of Misery-at-Risk



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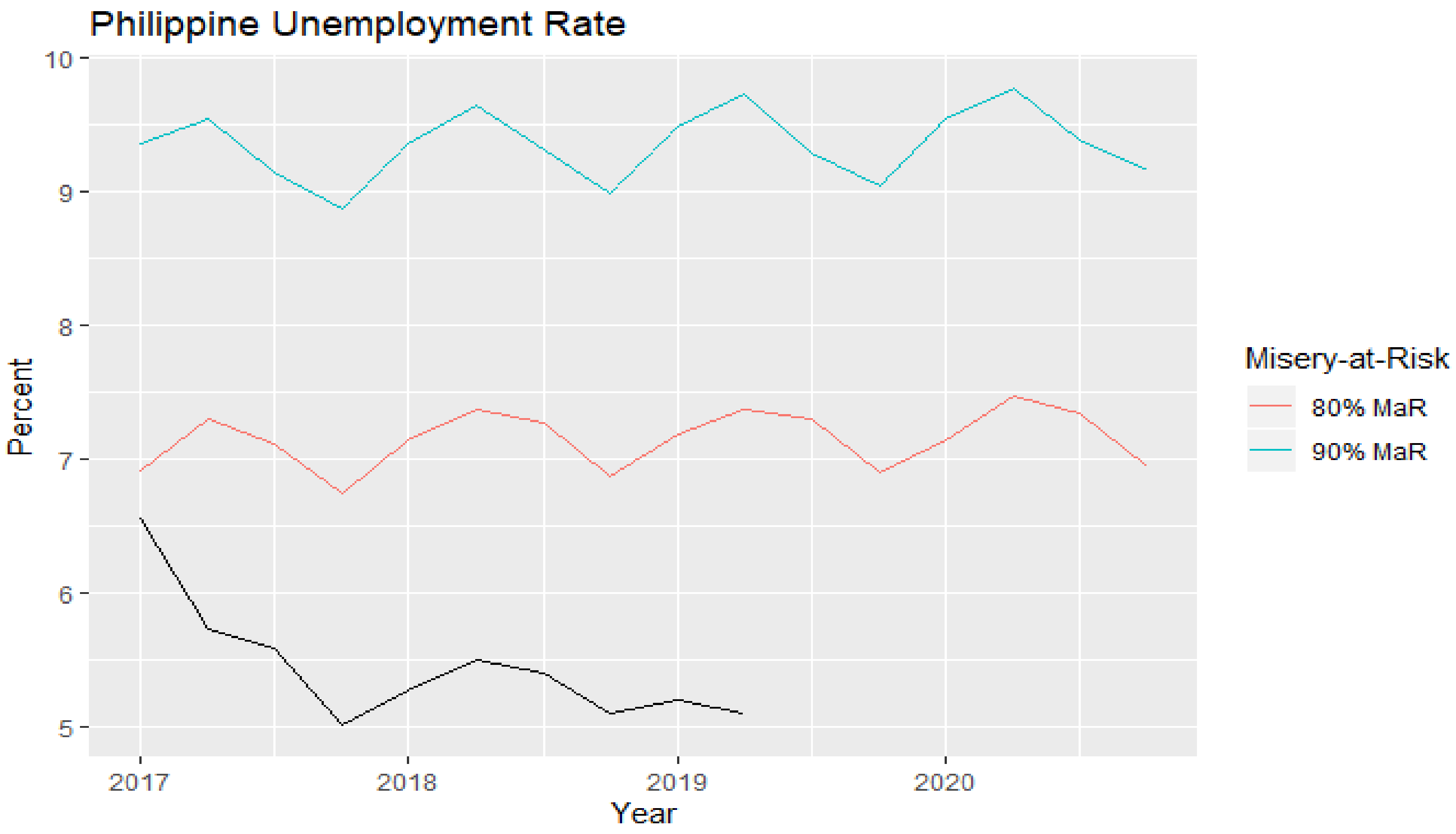
Hold-Out Analysis of Misery-at-Risk



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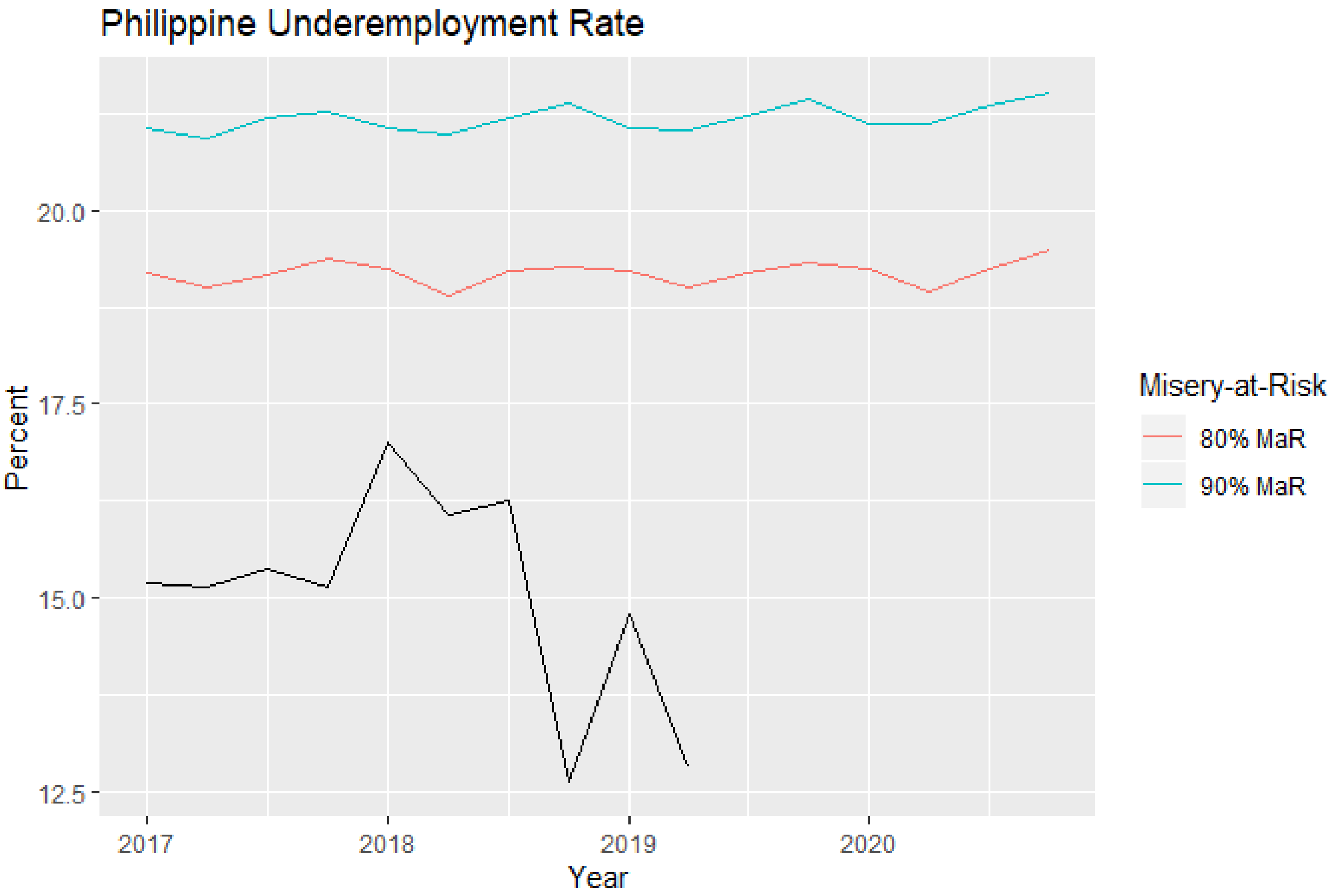

Summary and Conclusion

The aim: propose a methodology that will aid in the monitoring of the economic situation of the country through the index of misery as measured by inflation and job quality and availability.

The misery-at-risk was devised with this aim in mind, and used principles of modern developments in time series and data science for its inception.

This research seeks to open more opportunities in devising economic monitoring approaches by adopting methods from other fields such as finance and data science.

Thank you very much and G’Day!